Chapter 5

NEWTON'S LAWS

A.) Newton's Three Laws:

1.) Newton's First Law: In an inertial frame of reference, bodies in motion tend to stay in motion in a straight line, and bodies at rest tend to stay at rest, unless impinged upon by a net external force.

a.) Example: When an object in space is given a quick push and then left alone, it will move with a constant velocity in the direction of the push until an outside (external) force accelerates it into another kind of motion (i.e., makes it go faster, slower, change directions, or some combination thereof).

Until a net external force is applied, the constant-velocity, straight-line motion of a body will continue unchanged.

2.) Newton's Second Law: The *acceleration* $\boldsymbol{a}$ of a body (as a vector) is proportional to the *net force* $\boldsymbol{F}$ (also a vector) acting on the body.

a.) Mathematically, this can be stated as

$$\mathbf{F} = m\mathbf{a},$$

where the proportionality constant *m* is the *mass of the object being accelerated* (see section on "What is MASS" at the end of this chapter).

b.) As a force in the *x direction* will not make a body accelerate in the *y direction*, we can break both the *net force* and *acceleration* into their component parts and write three direction-related force equations:

$$F_{net,x} = m\,(a_x),$$
$$F_{net,y} = m\,(a_y),$$
$$F_{net,z} = m\,(a_z).$$

Note 1: These look like scalar equations because we haven't included unit vectors to designate direction, but each nevertheless represents a vector quantity. As such, we need to be careful of *direction* as denoted by *positive* and *negative* signs.

Note 2: Newton's Second Law is the work-horse of the three laws. It is a very powerful tool for analyzing situations in which forces and accelerations are involved. There is a technique involved in its use, outlined in *Part B*. LEARN THE APPROACH. Understand it. You should become so familiar with it that you can use it to evaluate problems *you have never seen before* using nothing more than it and your head.

3.) <u>Newton's Third Law:</u> For every force in the universe, there exists somewhere an *equal* and *opposite* reaction force.

Note: This idea of an *action/reaction* pair is somewhat misleading as it suggests that one force follows the other. As the examples will show, that is not the case. We will use the terminology because it is standard, not because it is an intelligent way of characterizing the interaction.

a.) Example 1: Imagine hitting a door with your hand. Doing so applies a force to the door which, if great enough, will break the door. In turn, the door applies a force to you which, if great enough, will break your hand. The size and direction of the force you apply must, according to N.T.L., be equal and opposite the force it applies to you.

i.) In the case of the door, the *action* is characterized by the statement, "You apply a force to the door" while the *reaction* is characterized by the statement, "The door applies a force to you." Notice that the wording is almost identical with the exception of reversed noun and/or pronoun order. This is always the case with the characterization of action/reaction pairs.

ii.) Don't be misled by the apparent personification of "the door." It obviously hasn't made a conscious decision to hit you back, even though the characterization implies that such be the case. Newton's Third Law is a commentary on the *structural reality* of the universe. The only way forces can be generated are in action/reaction pairs. By implication, if you try to apply 180 pounds of force to a table that can only provide 150 pounds of reaction force back on you, you will never succeed. The table will give way as soon as you exceed the 150 pounds that it can apply to you.

b.) Example 2: If the *earth* applies a gravitational force on *you*, what is the reaction force that must, according to N.T.L., exist?

i.) Answer: The reaction force is the gravitational force that *you* exert on the *earth*. Again, the same rearranged language is used to state the *reaction force* as was used to state the *action force*.

c.) Example 3: A truck runs head-on into a train. The train is ten times more massive than the truck. How do the forces experienced by each compare?

i.) The answer is, THEY ARE THE SAME! The train applies a force to the truck and, as a consequence, experiences an acceleration. The truck applies a force to the train and, as a consequence, experiences an acceleration. The two accelerations are different but, according to N.T.L., the two forces *must be* equal. Figures 5.1a and 5.1b depict this situation along with the fact that as *ma* equals the force on each body, that *product* must be the same for each body.

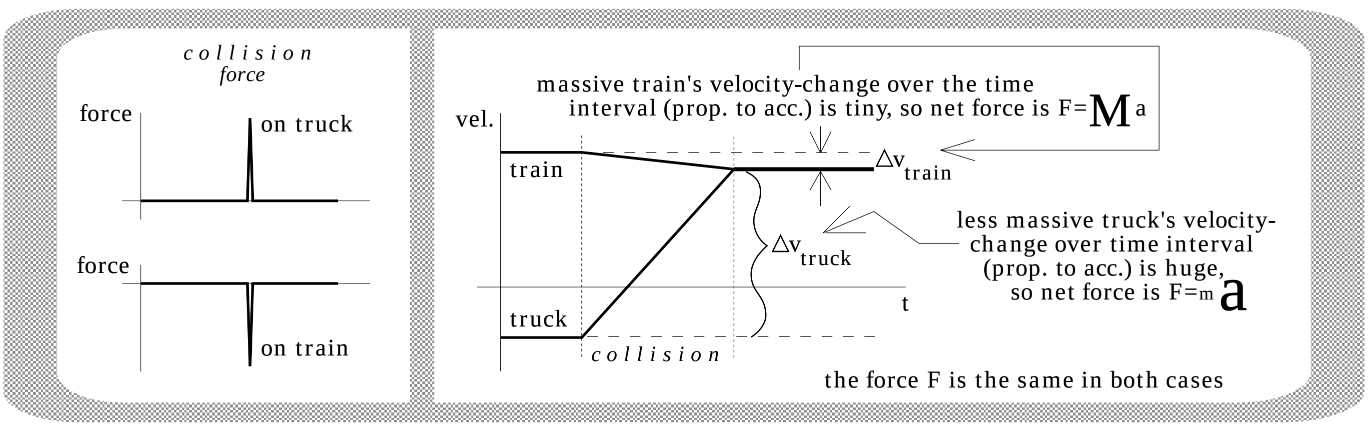

FIGURE 5.1a

FIGURE 5.1b

B.) Newton's Second Law and Types of Forces:

1.) *Newton's Second Law* is intimately related to force acting on a body. A brief preliminary discussion of the five kinds of forces you will work with follows in *Parts 2* through *6*:

2.) Gravitational force:

a.) Gravity is a force of attraction between any two massive objects.

b.) When the earth is one of the two bodies involved:

i.) The gravitational force felt by the second object (you, for instance) while positioned on the earth's surface will always be directed toward the earth's center; and

ii.) The gravitational force on the body will have a magnitude equal to the product *mg*, where *m* is the mass of the object in question and *g* is the gravitational acceleration near the earth (9.8 m/s^2 in the MKS system of units; 980 cm/s^2 in the CGS system of units; 32.2 ft/s^2 in the English (i.e., our) system of units).

Note: There are two metric systems of units (the MKS and the CGS systems) and one non-metric system (our own--the "English" system):

The MKS system uses *meters* for length, *kilograms* for mass, and *seconds* for time (hence, MKS). Force is in *newtons* and energy is in *joules*.

The CGS system uses *centimeters* for length, *grams* for mass, and *seconds* for time (hence, CGS). Force is in *dynes* and energy is in *ergs*.

The English system will never be used in this book (a base-10 version is used in engineering applications, but physicists generally stay away from it). For completeness, though, it uses *feet* for length, *slugs* for mass, and *seconds* for time. Force is in *pounds* and energy is in *foot-pounds*.

NEVER MIX UNIT-SYSTEMS. If you find yourself with mass information in grams (the CGS system) and you want to know the body's weight, do not multiply the mass by 9.8 m/s^2--the MKS value for the acceleration of gravity. Either convert the mass to kilograms (divide by 1000) or use the CGS value for *g* (i.e., 980 cm/s^2).

iii.) The gravitational force will be equal to the body's weight (that's right, folks, when you step on a scale and it measures your weight, you are really measuring *mg*--the force of attraction between you and the earth). As weight is a *force*, its units in the MKS system are *kilogram·meter/second*^{2}, or *newtons*.

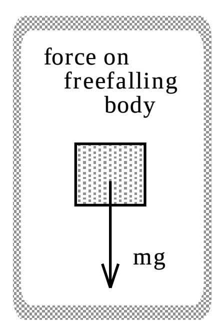

FIGURE 5.1c

c.) Example: Assuming we neglect air friction, a freely falling object has only one force acting on it--gravity (see Figure 5.1c).

3.) Normal force:

a.) A normal force is a force of support provided to a body by a surface in which the body is in contact;

b.) Normally characterized by *N*, a *normal force* is always directed *perpendicular to* and *away from* the surface providing the support;

c.) Example 1: In Figure 5.2a, a book is supported by a table. Figure 5.2b depicts all the forces acting on the book (this is called a *free body diagram*, or *f.b.d.*). Notice that the *normal force* acting on the book is *perpendicular to* and *away from* the table's top (the *gravitational force* is also shown).

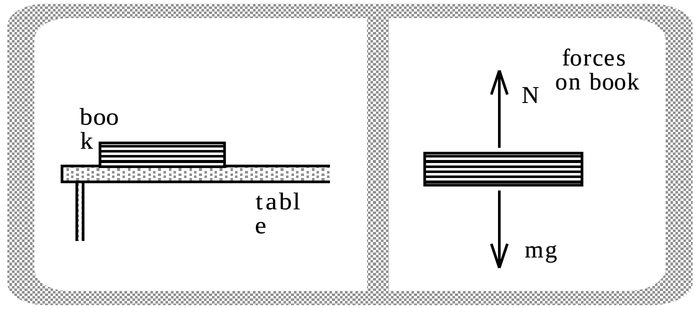

FIGURE 5.2a FIGURE 5.2b

d.) Example 2: In Figure 5.3a, a book rests on an incline plane. Figure 5.3b shows the f.b.d. for the forces acting on the book. As was the case in Example 1, the *normal force* acting on the book is *perpendicular to* and directed *away from* the incline's top, even though the top is not in the horizontal!

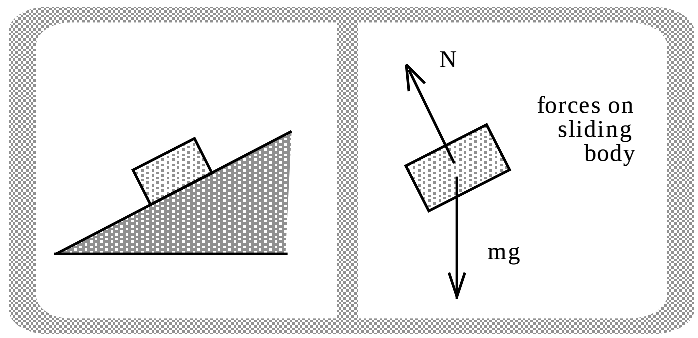

FIGURE 5.3a FIGURE 5.3b

Note: The *normal force* acts on the underside of the book, as shown in Figure 5.4. Though technically dubious, *normal forces* are usually presented as in Figure 5.3b. This is done for convenience (you will see why shortly). Either positioning is acceptable.

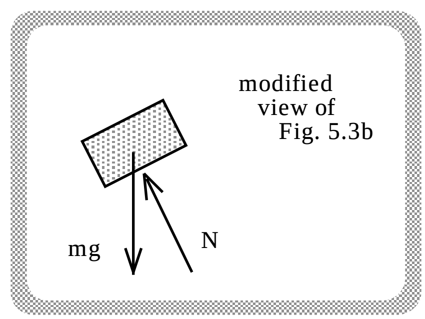

FIGURE 5.4

4.) <u>Tension force:</u>

a.) Tension force is applied to a body by a rope, string, or cable.

b.) Normally characterized by T, tension forces are always applied *along the line of the cable* and *away from* the body in question.

c.) Example: A string is attached to a hanging mass, threaded over a pulley, and attached to a second mass sitting on a table (see Figure 5.5a). The forces on the *hanging mass* are shown in Figure 5.5b; the forces on the *tabled mass* are shown in Figure 5.5c. In both cases, the *tension force* is directed *away from the body* and *along the line of* the string.

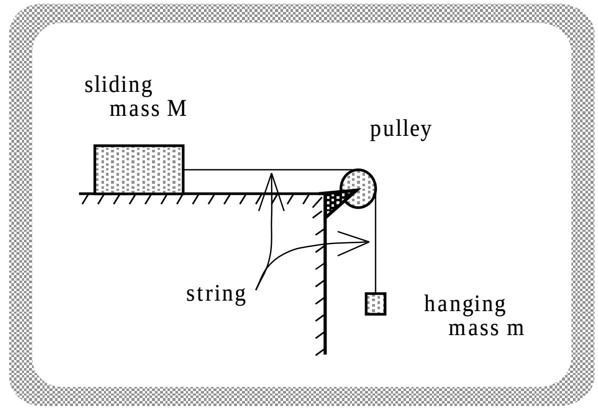

FIGURE 5.5a

Note: If the pulley is one of those mythical "frictionless, massless" jobs assumed-into-existence by physics departments across the country, the *massless* nature of the pulley will allow the *magnitude of the tension force* on either side of the pulley to be the same. In other words, an ideal pulley changes the DIRECTION of the tension force (in this case, from the horizontal to the vertical) but does not change the force's magnitude.

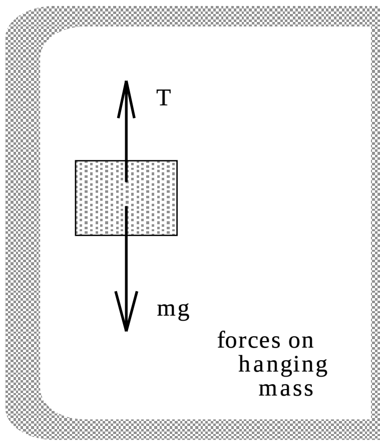

FIGURE 5.5b

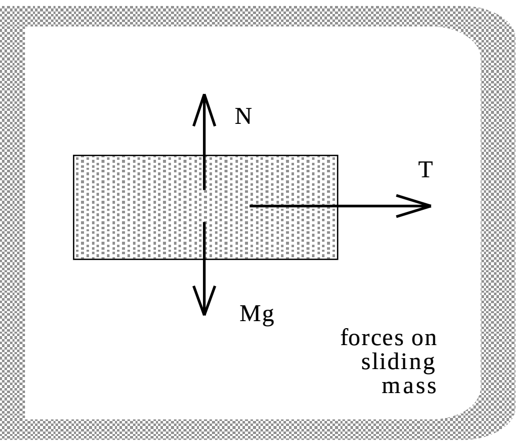

FIGURE 5.5c

5.) Frictional force:

a.) Friction is produced by the atomic interaction between two bodies as they either slide over one another (this is called *kinetic friction*) or sit motionless in contact with one another (this is called *static friction*).

Note: When two bodies come very close (i.e., rest against one another), there is a weak atomic bonding that occurs between the electrons of the one structure and the protons of the other (and vice versa). It is almost as though the atoms of the two bodies have melded to some degree. When the bodies try to move over one another, this bonding has to be sheared. That shearing is what produces the retarding effects we call friction.

b.) <u>Kinetic friction</u>: Sometimes called *sliding friction*, kinetic friction occurs when one body slides over or against a second body. The direction of *kinetic friction* is always *opposite* the direction of motion.

From experimentation, it has been observed that the amount of *kinetic friction* f_k a body experiences is proportional to the size of the *normal force* N exerted on the body by the structure it slides against. Mathematically, these two parameters (f_k and N) are related as:

$$f_k = \mu_k N,$$

where μ_k is a proportionality constant called the *coefficient of <u>kinetic</u> friction*.

As μ_k is normally given in a problem, all that is required to calculate the magnitude of the *kinetic frictional force* on a body is the *normal force* exerted by the supporting structure.

Note: The statement *kinetic friction is a function of normal force only--surface area has nothing to do with it* is true ONLY as long as you are dealing with two rigid bodies that are sliding relative to one another.

The tires of a dragster do NOT fit this bill. Dragster drivers spin their tires before a race to make them sticky. As such, the friction analogy for race tires is more like the dragging of a piece of Scotch tape across a desk than the sliding of rigid bodies--the bigger the surface area, the greater the stickiness and traction. My gratitude to Jim Malone (Mercersburg Academy) for contacting Mike Trinko of Goodyear's Race Group for this clarification.

c.) Example of *kinetic friction*: A block slides down a frictional surface (see Figure 5.6a). The *coefficient of kinetic friction* μ_k is known. Figure 5.6b shows all the external forces acting on the block.

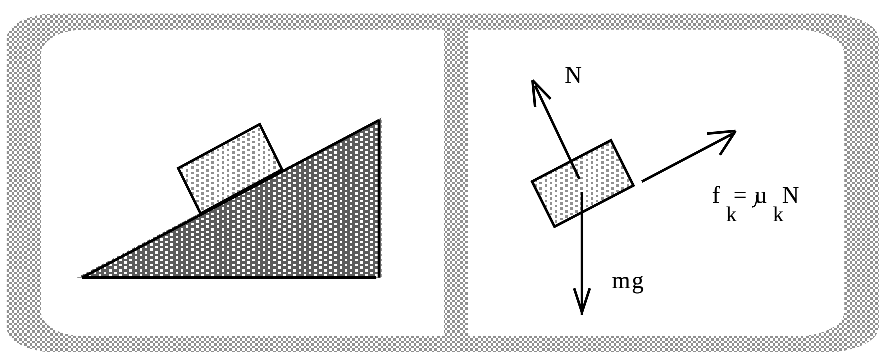

FIGURE 5.6a FIGURE 5.6b

Note: Notice that the normal force N sometimes equals the weight mg of the sliding body (an example of this can be seen in Figure 5.2b), BUT NOT ALWAYS. For the incline plane depicted above, N and mg are not even in the same direction.

d.) Static friction: If a book, for instance, sits on a table and a tiny external force is applied to the book *parallel to the table top*, the force may or may not move the book. There will be a force-of-opposition in this case provided by *static friction*. Its cause is the same as the partial atomic bonding that produces *kinetic friction* with one exception; static frictional bonding is stronger (with the two surfaces stationary relative to one another, the atoms meld more deeply requiring a greater shearing force to separate them).

If successively harder and harder forces are applied, the static frictional force will counter each push with a force of equal and opposite magnitude until the *applied force* is great enough to shear the partial bonding between the surfaces. At that point, the book will break loose and begin to slide.

It has been experimentally observed that this *maximum* static frictional force $f_{s,max}$ is proportional to the *normal force* N applied to the book by the table. Mathematically, these two parameters ($f_{s,max}$ and N) can be related as:

$$f_{s,max} = \mu_s N,$$

where μ_s is called the *coefficient of static friction*.

Note: Most physics books do not write the *maximum static frictional force* as $f_{s,max}$. Instead, they simply write f_s. For simplicity, we will do the same. Be clear, though. If you know f_s, you know only *one of the infinitely many* possible static frictional forces that could be exerted between the two bodies. Which force do you know? The *maximum static frictional force*.

On the other hand, if you know f_k you know the single, CONSTANT *kinetic frictional force* that exists between two bodies sliding relative to one another. No matter what their velocity (assuming heating doesn't change the characteristics of the two surfaces and alter the *coefficient of kinetic friction*), the *frictional force* f_k will always be the same.

The two quantities--f_s and f_k-- look similar as far as notation goes, but they tell us two different things!

6.) Push-me, pull-you force:

a.) Any force that does not fall into one of the above categories falls into this one. Ex: A shove from a friend. The magnitude of such a force is usually characterized by an F, possibly with a subscript.

C.) Newton's Second Law--APPROACHES:

1.) There are two ways to deal with Newton Second Law problems.

a.) The first is the formal, technically kosher way to proceed. It has specific steps and works on even the most convoluted force/acceleration problems. These include situations in which forces do not act in the same direction as the motion (e.g., centripetal situations), situations in which several objects within the system experience non-equal accelerations, and, with slight modification, situations in which some of the forces involved are velocity dependent.

b.) The second approach is a simplified version of the first. When applicable, it makes the evaluation of N.S.L. problems considerably quicker and easier (this can be very useful when dealing with multiple choice AP questions). As such, your initial temptation may be to focus on that approach to the exclusion of the first. That would be a serious mistake. Getting the answer isn't important, here. Understanding the approaches is. To that end, both approaches will be presented in the evaluation of the example problems in the next section.

Note: The *formal approach* is the only approach presented in the Solutions section at the back of the book. If you have correctly used the simplified version on a given problem, you can still use the book-end solutions to check your final answer.

2.) The simplified approach: When dealing with the simplified approach, the question you want to ask is, "What are the forces that motivate or retard acceleration in the system?" Determine those forces, equate them to ma, where m is the total mass of the system, and you're done.

3.) The formal approach: The following is a step-by-step outline for the use of the formal approach associated with Newton's Second Law as a problem-solving tool. Read through quickly, then continue on to see how the technique is used in the Example Problems section. Once you have read through that section, use this outline as a reference whenever you tackle complex Newton's Second Law problems.

a.) Look at the problem, blanch, think *I couldn't possibly do this*, then begin *the approach* starting with *Step b*.

b.) Draw a sketch of the entire system if one is not provided.

c.) Take one body in the system and draw a *free body diagram* (f.b.d.) for it.

i.) A *free body diagram* is a sketch of the body-in-question (it is normally depicted as a box) showing *all the forces acting on the body*. These forces are depicted as arrows. They don't have to be drawn to scale but do have to be directionally accurate (i.e., if the force is in the vertical, don't draw it halfway between the vertical and the horizontal).

Note: F.B.D.'s show only the forces that act *on the body-in-question*. They do *not* show forces that the body applies to other bodies.

ii.) When drawing an f.b.d., be sure the *orientation* of the box representing the body is the same as the actual orientation of the body in the problem. That is, if the object is a crate on an incline, the f.b.d. should depict the object as a box *on a slant*, not as a box in the horizontal--see Figure 5.6.

d.) Choose x and *y axes* and place them on your f.b.d. One axis must be in *the direction of the acceleration you are trying to determine.*

Note: This means your axes will not always be in the vertical and horizontal. Think about the direction of acceleration of the crate sketched in Figure 5.6.

e.) If there are forces on the f.b.d. that are not along the x and y directions, replace them with their x and *y components*.

f.) Using N.S.L., sum the forces in the x direction and set them equal to the product ma_x. If an additional relationship (equation) is needed, sum the forces in the y direction and set them equal to the product ma_y (assuming you are interested in the variables involved).

g.) Repeat the above process for all the bodies in the system *or until you have enough equations to solve for the unknowns you are interested in* in the system.

WARNING: ASSUME NOTHING that can be derived using an f.b.d. and N.S.L.

D.) Example Problems:

The easiest way to become comfortable with both the simplified and formal approaches outlined above is to try a problem. There are two in this section: one that is relatively simple and a second that is more complex. Both have within them potential pitfalls. Look for the sticking points. Understanding these difficulties will make it easier for you to do other N.S.L.-type problems later on your own (this is especially true of the formal approach).

Note: While you are reading, remember that your goal is to generate equations that relate the *acceleration* of one or more of the masses within the dynamic (i.e., moving) system to the forces acting on that mass.

1.) Example 1--The Run-Away Car using the Formal Approach: Somebody gives your car a shove, then exits. The car rolls freely on a horizontal road toward a cliff. You try to stop the car by applying a known force $\boldsymbol{F}$ at a known angle θ (see Figure 5.7). If the car's mass is m, derive a general algebraic expression for the *acceleration* of the car and, for the fun of it, the *normal force* applied to the car by the ground. We'll use *the formal approach* first.

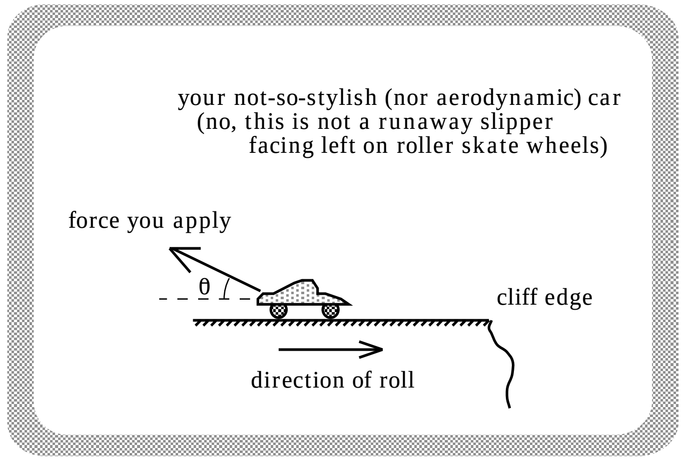

FIGURE 5.7

--Solution and Approach *By-the-Numbers*:

--Step a: The sketch is seen in Fig. 5.7.

--Step b: A *free body diagram* (f.b.d.) of the forces acting on the car is shown in Figure 5.8.

Notice that the *direction of the car's motion* (i.e., the direction of its velocity vector) is not the same as the *direction of its acceleration*. That means, simply, that the car is slowing down.

Notice also that the *velocity vector* is not found on the f.b.d. Only FORCES go on free body diagrams.

--Step c: Figure 5.9 shows the placement of appropriate coordinate axes. One is in the horizontal along the car's *line of acceleration*, and the other is *perpendicular* to the first.

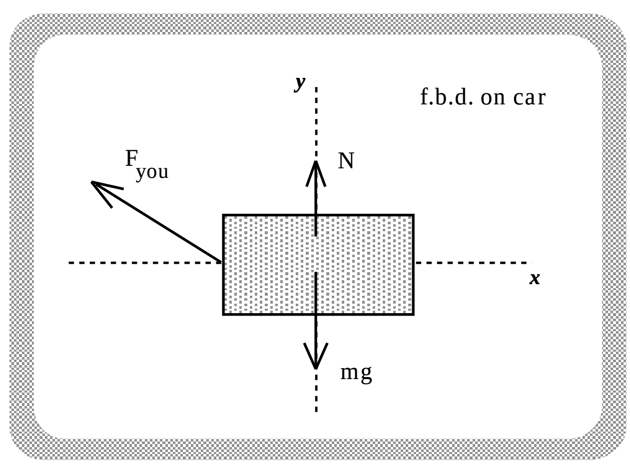

FIGURE 5.9

Note: Although it is customary to make horizontal axes *"+" to the right* and vertical axes "+" *in the upward direction*, you can define any direction as positive.

–Step d: There is one force that is off-axis--the pulling force F. Figure 5.10 shows a revised f.b.d. with F replaced by its components along the two axes.

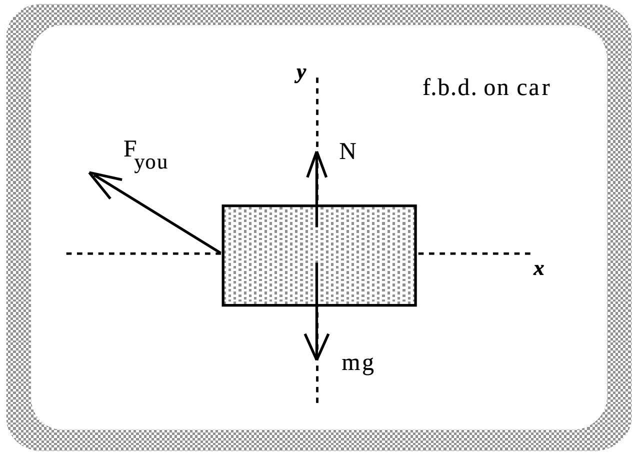

FIGURE 5.9

Note: In Figure 5.10, the vertical component of $\boldsymbol{F}$ (i.e., $F \sin\theta$) is positioned out away from the body, but it actually acts at the same point as does $\boldsymbol{F}$. It has been drawn as part of a right triangle to make its determination easier. Having made that determination, you could re-position the vector as shown in Figure 5.11. There is nothing wrong with doing this, though it does seem a waste of time if its original positioning doesn't confuse you.

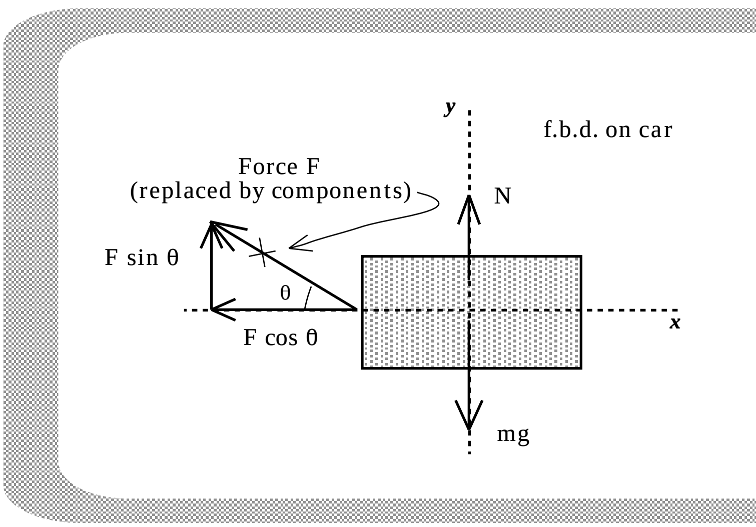

FIGURE 5.10

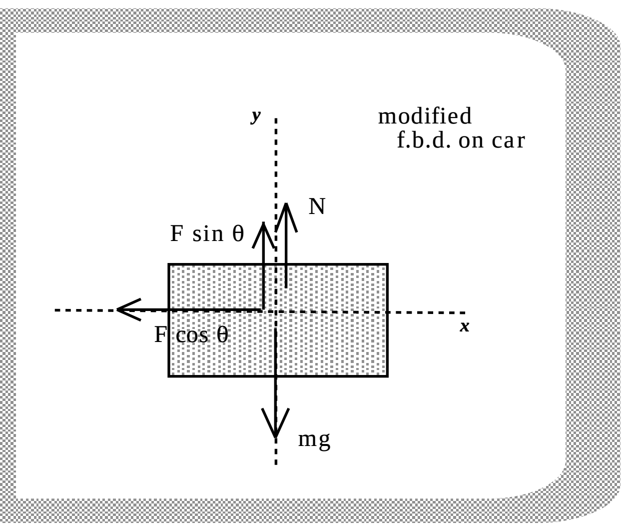

FIGURE 5.11

–Step e: The *sum of the forces* in the x and y directions is shown below. Notice that each has been preceded by a blurb that denotes what is about to happen. That is, the "$\underline{\Sigma F_x}$" notation alerts the reader that you are beginning a *sum-of-the-forces-equals-"ma"* process in a given direction. *You will be expected to use this same notation* as a preamble whenever you use Newton's Second Law.

$\underline{\sum F_x:}$

$$-F(\cos\theta) = -m(a_x)$$

$$\Rightarrow \quad a_x = F(\cos\theta)/m.$$

Note 1: Notice the *negative sign* in front of the *component of F along the x axis* term? If that force component had been in the opposite direction, its magnitude would have remained $F(\cos\theta)$, but the sign in front of the component would have been *positive*. In other words, the expression $F(\cos\theta)$ stands for the *magnitude* of the force. When writing out the N.S.L. expression, you have to account for the *direction* of a force by manually inserting the appropriate *positive* or *negative* sign in front of its magnitude value.

Note 2: Just as was the case with the force component discussed in Note 1, the acceleration term a_x can be treated in one of two ways. We can either define it to be a vector, leaving its sign embedded within the symbol, or we can define it to be a *magnitude*, unembed its sign, and manually put that sign in front of *ma*. For reasons that will become obvious later, the latter is preferable.

Note 3: This *unembedding the sign* isn't as scary as it looks. If the assumed direction for the acceleration is incorrect, we will simply end up with an acceleration value that has a negative sign in front of its magnitude when numerically calculated. As magnitudes cannot be negative, the negative sign simply means we have assumed the *wrong direction* for the acceleration.

–back to Step e:

In the *y direction*:

$\underline{\sum F_y:}$

$$+F(\sin\theta) + N - mg = +m(a_y) \qquad (= 0 \text{ as } a_y = 0).$$

As shown, the acceleration a_y is zero (the car is not hopping up and down off the road). As such, this expression can be re-written as:

$$N = -F(\sin\theta) + mg.$$

Note: The normal force exerted by the ground on the car is *not equal to the weight of the car*. This shouldn't be surprising. The car is being pulled upward by the vertical component of ***F***, requiring less *normal force* from the ground to keep the car from falling through the road.

--Step f: We have enough equations to solve for the parameters in which we are interested.

Using the derived equations with a car mass of 1000 kg, we find that if the angle is 30^o and the pulling force is 800 nts (you *are* a gooorilla):

$$
\begin{aligned}
a_x &= F(\cos\theta)/m \\
&= (800\text{ nt})(\cos 30^o)/(1000\text{ kg}) \\
&= .69\text{ m/s}^2
\end{aligned}
$$

and

$$
\begin{aligned}
N &= -F(\sin\theta) + mg \\
&= -(800\text{ nt})(\sin 30^o) + (1000\text{ kg})(9.8\text{ m/s}^2) \\
&= 9400\text{ nts.}
\end{aligned}
$$

Note: One *newton* is equal to about a quarter of a pound.

2.) Example 1--The Run-Away Car using the Simplified Approach: Somebody gives your car a shove, then exits. The car rolls freely on a horizontal road toward a cliff. You try to stop the car by applying a known force *F* at a known angle θ (see Figure 5.7). If the car's mass is *m*, derive a general algebraic expression for the *acceleration* of the car.

a.) The question to ask is, "What are the forces that motivate the car to accelerate?" In this case, the only force acting along the line of acceleration is $F(\cos\theta)$. Putting that equal to ma_x yields:

$$
\begin{aligned}
&F\cos\theta = ma_x \\
&\Rightarrow\ a_x = (F\cos\theta)/m.
\end{aligned}
$$

b.) Although this undoubtedly seems wickedly easy in comparison to the more plodding *formal approach* presented in the previous section, remember what we are doing. We are examining relatively simple problems with an eye to developing techniques that will serve you well when you are asked to take apart more complex scenarios. Be patient. As hard as it may be to believe, the *formal approach* is definitely the way to go in many situations.

3.) Example 2--Incline and Pulley using the Formal Approach: A string attached to a known hanging mass m_1 is threaded over a *frictionless, massless pulley* and attached at its other end to a second known mass m_2 that is supported by a *frictional* incline plane. The mass m_1 is additionally wedged *frictionlessly* against the incline by a known force F (see Figure 5.12). Both the angle of the incline θ_2 and the angle θ_1 at which F assaults the block are known. The *coefficient of kinetic friction* μ_k between m_2 and the incline is also known. Assuming the string is inextensible (that is, it isn't acting like a rubber band) and mass m_2 is moving *up* the incline when first viewed, determine the *magnitude* of the *acceleration* of the system.

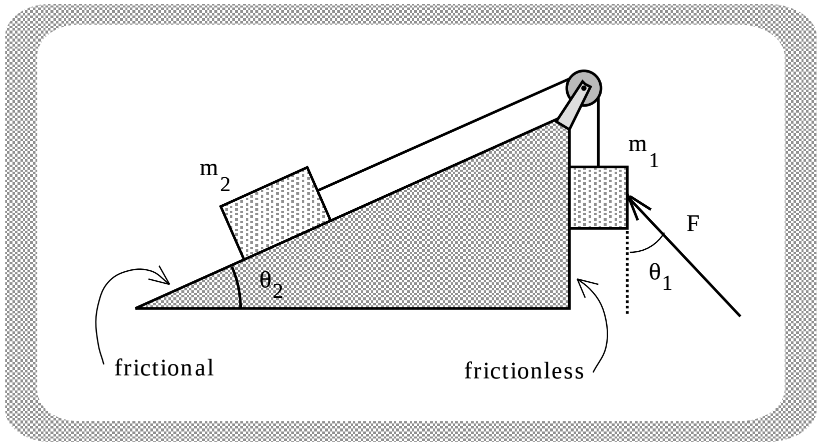

FIGURE 5.12

Note 1: I've done this analysis in steps to allow you to see how each of the formal steps alluded to in the previous section plays out in a problem. I'm doing it this way ONLY because the approach is new to you and I want you to see and understand what each formal step actually does. At the end of the presentation, I will redo the problem the way I would expect you to proceed if such a problem were given on a test.

Note 2: As the string is inextensible, the "magnitude of the acceleration of the system" will be the *magnitude* of the acceleration of either of the two masses. If we know a for one body, assuming a is a magnitude, we know it for the other.

--Solution and Approach *By-the-Numbers*:

--Step b: The sketch of the system has already been shown in Figure 5.12.

--Step c: We will begin with mass m_1. An f.b.d. of the forces acting on m_1 is shown in Figure 5.13.

Note: The *frictional force* between any two objects sliding relative to one another is $f_k=\mu_k N$. If there had been a *frictional force* acting between m_1 and the incline's vertical wall, we would need to determine N_1 before we could determine f_k. In this case, there is no frictional force along that surface so there is no need to determine N_1. Nevertheless, to be complete, our *free body diagram* MUST INCLUDE N_1 and all other horizontal forces acting on m_1.

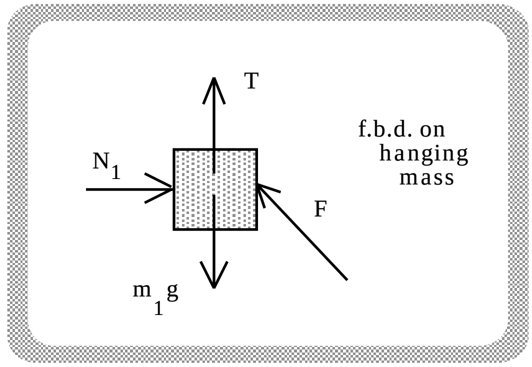

FIGURE 5.13

Bottom line: By definition, a *free body diagram* includes ALL THE FORCES ACTING ON THE BODY FOR WHICH IT IS DRAWN.

–Step d: Figure 5.14 shows the off-axis-forces broken into their components.

–Step e: The *sum of the forces* in the *v direction* is shown below.

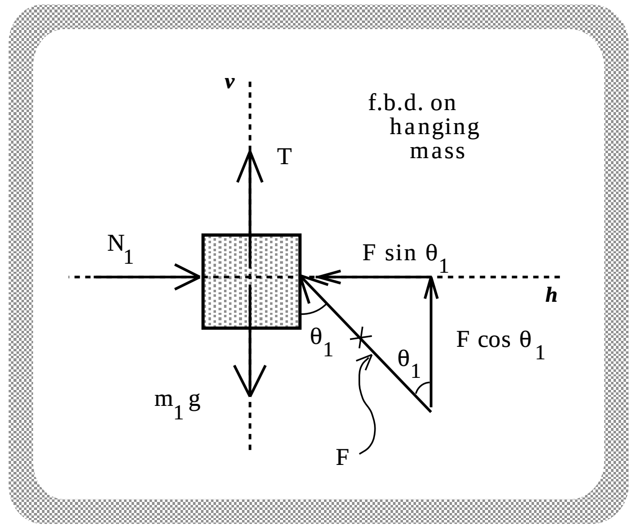

FIGURE 5.14

$\sum F_v$:

$$T - m_1g + F\cos\,\theta_1 = -m_1(a_v)$$
$$= -m_1a.$$

(a is defined to be the *magnitude* of the acceleration of both m_1 and m_2). This implies:

$$T = m_1g - F\cos\,\theta_1 - m_1a \qquad \text{(Equation A).}$$

Note 1: We know that the initial velocity of m_1 is *downward* (m_1 has to be moving downward if m_2 is moving *up* the incline), but we really don't know whether the *acceleration* on m_1 is *up* (i.e., m_1 is slowing) or *down* (i.e., m_1 is increasing speed). Not knowing the relative sizes of m_1 and m_2 makes it impossible to tell which is happening.

As can be seen from the work done above, we are doing this problem on the assumption that the acceleration of m_1 is downward, hence the *negative* sign in front of the m_1a term on the right-hand side of N.S.L. If we are wrong, it won't matter as long as we are *consistent throughout the problem*. That is, if we assume m_1 accelerated *downward,* we must also assume that m_2 accelerates *up the incline*.

As was noted some sections ago, when the mass values are put into the final equation, the *sign* of the calculated acceleration magnitude will either be *positive* or *negative*. If it is positive, it means we have assumed the correct direction for the acceleration; if it is negative, it means we have assumed the wrong direction. The magnitude will be the same in either case.

--**Step f:** We have one equation (Equation A) and two unknowns (*T* and *a*). We need another equation, which means we must look at m_2*'s* motion. Starting the approach over again on m_2:

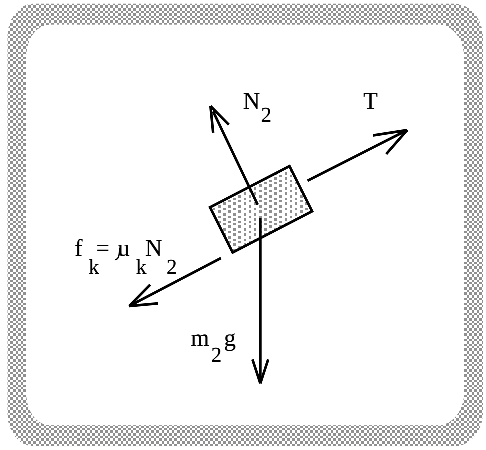

FIGURE 5.15

<u>For m_2:</u>

--**Step b:** The sketch of the system is still shown in Figure 5.12.

--**Step c:** A *free body diagram* of the forces acting on m_2 is shown in Figure 5.15. Notice the frictional force acts *down* the incline--opposite the *upward* motion of m_2.

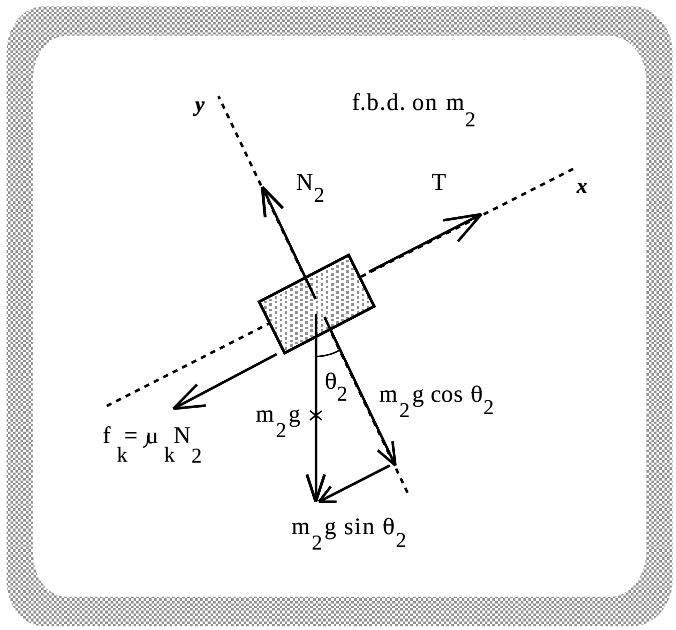

FIGURE 5.16

--**Step d:** Figure 5.16 shows the placement of appropriate coordinate axes. The *line* of m_2*'s* acceleration is along the incline, so one of the axes (labeled *x*) is placed along that line with the second axis (labeled *y*) *perpendicular* to the first.

--**Step e:** Summing the forces in the *y direction* will allow us to solve for N_2:

$\underline{\sum F_y}$:

$$N_2 - m_2g(\cos\theta_2) = m_2(a_y)$$
$$= 0 \qquad \text{(as } a_y = 0\text{)}$$
$$\Rightarrow \quad N_2 = m_2g(\cos\theta_2) \qquad \text{(Equation B).}$$

Now that we know N_2, we can sum the forces in the *x direction*:

$\underline{\sum F_x}$:

$$T - m_2 g (\sin\ \theta_2) - \mu_k N_2 = m_2(a_x)$$
$$= m_2 a,$$

(remember, a was defined as the magnitude of the acceleration of both m_1 and m_2). Substituting *Equation B* in for N_2 yields:

$$T - m_2 g(\sin\ \theta_2) - \mu_k\ [m_2 g(\cos\ \theta_2)] = m_2\ a.$$

Substituting *Equation A* in for T yields:

$$[m_1 g - F\cos\ \theta_1 - m_1 a] - m_2 g(\sin\ \theta_2) - \mu_k\ [m_2 g(\cos\ \theta_2)] = m_2 a$$

$$\Rightarrow \quad a = [m_1 g - F\cos\ \theta_1 - m_2 g(\sin\ \theta_2) - \mu_k m_2 g(\cos\ \theta_2)] / (m_1 + m_2).$$

--Step 6: Finito!

4.) Example 2: Incline and Pulley using the Formal Approach as expected ON A TEST:

For m_1: (f.b.d., axes, and components shown in Figure 5.14)

$\underline{\sum F_v}$:

$$T - m_1 g + F\cos\ \theta_1 = -m_1(a)$$
$$\Rightarrow \quad T = m_1 g - F\cos\ \theta_1 - m_1 a \qquad \text{(Equation A).}$$

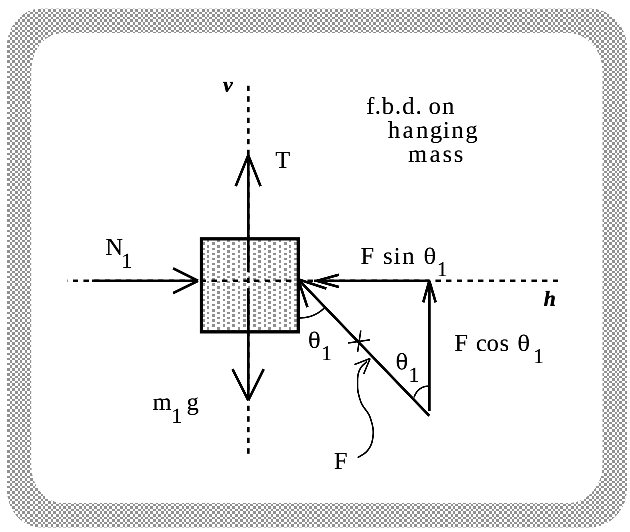

FIGURE 5.14

For m_2: (f.b.d., axes, and components shown in Figure 5.16)

$\underline{\sum F_y}$:

$$N_2 - m_2 g\ (\cos\ \theta_2) = m_2\ (a_y) \qquad (= 0 \text{ as } a_y = 0)$$
$$\Rightarrow \quad N_2 = m_2 g\ (\cos\ \theta_2) \qquad \text{(Equation B).}$$

$\underline{\sum F_x}$:

$$T - m_2 g\,(\sin\,\theta_2) - \mu_k N_2 = m_2(a_x)$$
$$= m_2 a,$$

Using *Equation B* to eliminate N_2 and *Equation A* to eliminate T, we can write:

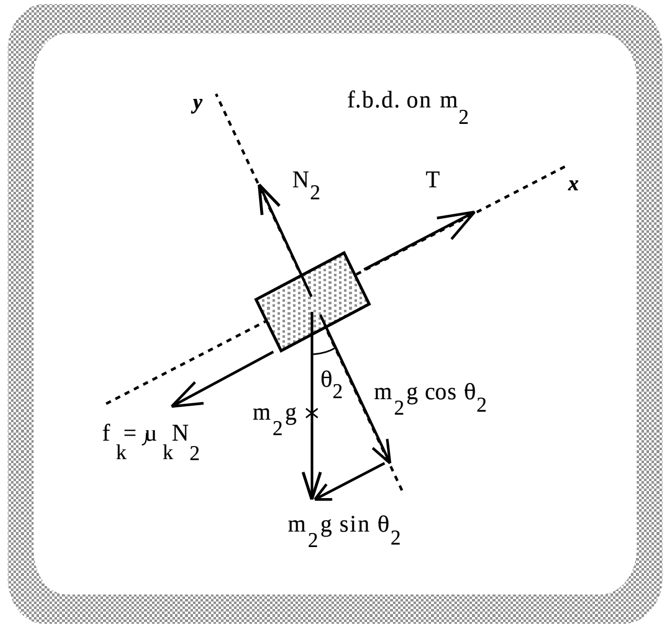

FIGURE 5.16

$$\begin{aligned} &T \qquad\qquad\qquad\quad - m_2g(\sin\,\theta_2) - \mu_k \qquad\quad N_2 \qquad = m_2a \\ &[m_1g - F\cos\,\theta_1 - m_1a] - m_2g(\sin\,\theta_2) - \mu_k\,[m_2g(\cos\,\theta_2)] = m_2a \end{aligned}$$

$$\Rightarrow \quad a = [m_1g - F\cos\,\theta_1 - m_2g(\sin\,\theta_2) - \mu_k m_2 g(\cos\,\theta_2)]\;/\;(m_1 + m_2).$$

5.) Observations about *Example Problem 2*:

a.) Let's try some numbers: Assume $m_1 = 5\ kg$, $m_2 = 8\ kg$, $\mu_k = .4$, the applied force $F = 3$ *newtons,* $\theta_1 = 50^o$, and $\theta_2 = 30^o$. Re-ordering, remembering that the magnitude of the *acceleration of gravity* $g = 9.8\ m/s^2$, and omitting units to save space, the calculation becomes:

$$\begin{aligned} a &= \big[[m_1g - m_2g(\sin\,\theta_2) - \mu_k m_2 g(\cos\,\theta_2)] - F\cos\,\theta_1\big]\;/\;(m_1 + m_2) \\ &= \big[g\,[m_1 - m_2(\sin\,\theta_2) - \mu_k m_2(\cos\,\theta_2)] - F\cos\,\theta_1\big]\;/\;(m_1 + m_2) \\ &= \big[(9.8)\,[(5\,) - (8\,)\sin 30^o - (.4)\,(8)\cos 30^o] - (3)\cos 50^o\big]\;/\;(5 + 8\,) \\ &= -1.48\ \text{m/s}^2. \end{aligned}$$

Note: We have a *negative sign* in front of the calculated acceleration. Why? In the problem, the hanging mass m_1 was stated to be traveling downward (that was the way the problem was set up). Additionally, we assumed that m_1 was *accelerating* downward (we didn't know for sure--that was our guess). What the *negative sign* means is that, given the mass-values used, m_1*'s acceleration* was not *downward* but *upward*. Evidently, the hanging mass moved downward, *slowing in the process*.

b.) If m_1 had been moving *upward*, what would have been different? The *free body diagram* for m_1 would have looked identical to that shown in Figure 5.13, but m_2*'s* f.b.d. would have been different (see Figure 5.17). We still wouldn't have known the acceleration's direction, so we still would have had to assume one. If we had taken it to be *up the incline*, the final acceleration expression derived from Newton's Second Law would have been:

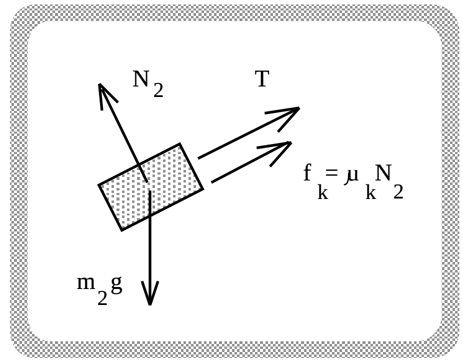

FIGURE 5.17

$$\begin{aligned} a &= \left[[m_1 g - m_2 g(\sin \theta_2) + \mu_k m_2 g(\cos \theta_2)] - F\cos \theta_1\right] / (m_1 + m_2) \\ &= [g\,[m_1 - m_2(\sin \theta_2) + \mu_k m_2(\cos \theta_2)] - F\cos \theta_1] / (m_1 + m_2) \\ &= \left[(9.8)\,[(5) - (8)\sin 30^o + (.4)(8)\cos 30^o] - (3)\cos 50^o\right] / (5 + 8) \\ &= +2.69 \text{ m/s}^2. \end{aligned}$$

Notice the acceleration value in this case turns out to be positive, implying that the assumed direction-of-acceleration for m_1 (<u>downward</u>) was correct (given the mass values we've used in the evaluation). Evidently, in this case, the hanging mass moved upward, *slowing in the process*.

Conclusion: In both cases, friction was great enough to slow the motion.

6.) <u>Example 2--Incline and Pulley using the Simple Approach:</u> A string attached to a known hanging mass m_1 is threaded over a *frictionless, massless pulley* and attached at its other end to a second known mass m_2 that is supported by a *frictional* incline plane. The mass m_1 is additionally wedged *frictionlessly* against the incline by a force F (see Figure 5.12). Both the angle of the incline θ_2 and the angle θ_1 at which F assaults the block are known. The *coefficient of kinetic friction* μ_k between m_2 and the incline is also known. Assuming the string is inextensible (that is, it isn't acting like a rubber band) and mass m_2 is moving *up* the incline when first viewed, determine the *magnitude* of the *acceleration* of the system.

a.) As before, the question to ask is, "What are the acting forces that motivate the system as a whole to accelerate?"

b.) In this case, there are several forces motivating the total mass of the system (i.e., $m_1 + m_2$) to accelerate. They are:

i.) The force of gravity on m_1: Its magnitude is m_1g and its effect via the string is to attempt to accelerate m_2 UP the incline.

Note: If we are talking about gravity on m_1, why do we care about m_2? We need some way to assign positiveness or negativeness to forces acting on *the SYSTEM* when, in fact, those forces may well be acting on different bodies oriented in entirely different ways from one another. The easiest way to do that is to pick a single body in the system and determine how a given force will ultimately affect that body. In this case, gravity is pulling m_1 down. This will tend to pull m_2 UP the incline, so I will arbitrarily identify that effect to be associated with a positive force.

ii.) The component of F that pushes m_1 upward: Its magnitude will be $F\cos\theta_1$. Its effect is to attempt to accelerate m_2 DOWN the incline, so we will call it a negative force.

iii.) The component of gravity that pushes m_2 DOWN the incline: Its magnitude is $m_2g\sin\theta_2$ (this is the component of gravity *along the line of the incline*), and we will call it a negative force.

iv.) The force of friction on m_2: With the normal force equal to $m_2g\cos\theta_2$, friction's magnitude is $f_k = m_kN = m_k(m_2g\cos\theta_2)$. The direction of the frictional force will be opposite the direction of motion, or DOWN the incline, making it a negative force.

v.) The temptation might be to add in the tension force. In this case, tension is an internal force--a force that exists as a consequence of the interaction of pieces of the system. Internal forces will not affect the overall motion of the system. That is, if we add the tension force on m_1 (this will be upward on m_1 motivating m_2 to accelerate down the incline) to the tension force on m_2 (this will motivate m_2 to accelerate up the incline), the net effect will be zero. As such, tension IN THIS CASE is ignored.

c.) Summing up all of the forces and putting them equal to the total mass in the system, we get:

$$m_1g - F(\cos\theta_1) - m_2g(\sin\theta_2) - \mu_k(m_2g\cos\theta_2) = (m_1 + m_2)a$$
$$\Rightarrow \quad a = [m_1g - F\cos\theta_1 - m_2g(\sin\theta_2) - \mu_k m_2g(\cos\theta_2)] / (m_1 + m_2).$$

Note: AGAIN, don't get too excited about all of this. The simplified approach isn't always going to work. Get to know both procedures!

E.) Friction and Free Fall:

1.) Consider a body of mass *m* free falling in a fluid from rest under the influence of gravity.

a.) For pure free fall with no fluid-produced friction, theory predicts that the body's acceleration will be equal to g and the body's velocity at any point in time will (according to the kinematic equations) be equal to $v = -g\Delta t$ (this comes from $v = v_1 + (-g)\Delta t$, with $v_1 = 0$ and Δt being the *time of flight*).

b.) The situation changes dramatically when friction is taken into account.

2.) Friction in a free fall situation is caused by molecules colliding with the body as it falls through the fluid. The faster the object travels, the more molecules are hit *per unit time* and the more retarding force is generated. When this *frictional force* completely counterbalances gravity, the net force equals zero and the body *stops accelerating* (i.e., its velocity does not change from then on). Once the body reaches that state, it is said to be traveling at "terminal velocity."

3.) Assume a frictional force that is *proportional to the magnitude of the velocity* of the body (i.e., $f \propto v$, which is to say $f = kv$, where k is a proportionality constant related to the density of the material through which the body travels). We want to determine two things: the body's *terminal velocity* and the body's *velocity as a function of time.*

Minor Note: If we had been dealing with *air friction*, the frictional force would have been better approximated as $f = kv^2$.

4.) The first of the two questions is relatively easy:

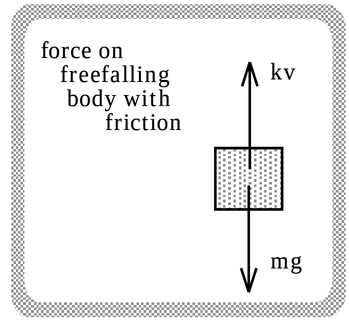

FIGURE 5.18

a.) Figure 5.18 shows the situation. The body accelerates until the frictional force is exactly equal and opposite the force of gravity. At that point, the acceleration becomes zero and we can use N.S.L. to state:

$$\underline{\sum F_y:}$$

$$kv_{term} - mg = m(a)$$
$$= 0 \qquad (\text{as "a = 0"})$$

Dividing both sides by *k* yields the maximum velocity the body will attain. This *terminal velocity* is $v_{term} = (mg)/k$.

5.) Determining the body's *velocity as a function of time* is a considerably more difficult process. (Note that for simplicity, *v(t)* will be written from here on as *v*).

a.) Begin by writing N.S.L. in its most general form for the forces acting on the body at an arbitrary point in time *t*:

$$\underline{\sum F_y:}$$

$$kv - mg = -m(a)$$
$$= -m[dv/dt].$$

IMPORTANT Note: The sign selection in front of the *ma* term isn't as obvious as one might expect. <u>READ *Section I*</u> at the end of the chapter to understand the problem and its solution.

b.) We need to manipulate this equation to get both velocity-related terms (i.e., both *v* and *dv/dt*) on the same sides of the equal sign. By dividing both sides by *-m*, then rearranging, we get:

$$\frac{dv}{dt} = -\left[\frac{k}{m}v - g\right].$$

c.) We need to link the *v* and *dv* term by multiplication or division (the reason for this will become evident shortly). To do so, we rewrite our equation as:

$$\frac{dv}{dt} = \left(-\frac{k}{m}\right)\left[v - \frac{mg}{k}\right].$$

Note: No, you probably wouldn't have thought to do this if you were on your own. Then again, if you already knew the tricks of the trade you would not need to take this class!

d.) The above expression states: The *time rate of change of velocity* equals a *mass related constant* times a *velocity related quantity*.

e.) We are now in the position to divide both sides by $v - mg/k$ and multiply both sides by dt (ah, the sound of mathematicians groaning everywhere--it's OK, though, the equation that follows is true and good if the equation above is assumed). Doing so yields:

$$\frac{dv}{\left[v - \frac{mg}{k}\right]} = \left(-\frac{k}{m}\right)dt.$$

f.) The above equation states: A *differential change in velocity dv* divided by a *velocity dependent quantity* evaluated at time t (where t is the time about which dt straddles), is equal to a *constant* times the size of the *differential time interval dt* over which the velocity change occurred.

Fortunately for those not into semantics, what the equation *says* is not nearly as important as what we can do with it.

g.) If we assume the body's velocity at $t = 0$ is $v = 0$, we can integrate both sides of this equation (i.e., summing the *differential velocity-related-quantities* over the time interval, etc.) between $t = 0$ and t, yielding:

$$\int_{v=0}^{v(t)} \frac{dv}{\left[v - \frac{mg}{k}\right]} = \left(-\frac{k}{m}\right)\int_{t=0}^{t} dt.$$

h.) The integral $\int dt$ simply equals t evaluated at $t = 0$ and t. The integral $\int dv/(v + c)$, where c is a constant, equals the *natural log* of the absolute value of $v + c$, evaluated at the limits. As such, we can write:

$$\int_{v=0}^{v(t)} \frac{dv}{\left[v - \frac{mg}{k}\right]} = \left(-\frac{k}{m}\right)\int_{t=0}^{t} dt$$

$$\Rightarrow \quad \ln\left|v - \frac{mg}{k}\right|_{v=0}^{v(t)} = \left(-\frac{k}{m}\right)t.$$

i.) Noting that *v(t)* is always smaller in magnitude than the terminal velocity of *mg/k*, the argument *v - mg/k* will always be negative. That means we can remove the *absolute value* sign in the *natural log* expression as long as we re-write the argument as *mg/k - v(t)* (this will make that difference *always positive*, as required by the *absolute value sign* we wanted to remove). Doing so, then evaluating, yields:

$$\ln\left|v-\frac{mg}{k}\right|_{v=0}^{v(t)} = \left(-\frac{k}{m}\right)t$$

$$\Rightarrow \quad \ln\left(\frac{mg}{k}-v\right)_{v=0}^{v(t)} = \left(-\frac{k}{m}\right)t$$

$$\Rightarrow \quad \left[\ln\left(\frac{mg}{k}-v(t)\right)-\ln\left(\frac{mg}{k}-0\right)\right] = \left(-\frac{k}{m}\right)t.$$

j.) The expression *ln (a) - ln (b)* equals *ln (a/b)*. As such, our expression can be re-written as:

$$\ln\left[\frac{\left(\frac{mg}{k}-v(t)\right)}{\frac{mg}{k}}\right] = \left(-\frac{k}{m}\right)t.$$

k.) We need to get the velocity variable out from under the *"ln" function*. As $e^{\ln(a)} = a$, we can use both sides of the equation as exponents of *e* and write:

$$e^{\ln\left[\frac{\left(\frac{mg}{k}-v(t)\right)}{\left(\frac{mg}{k}\right)}\right]} = e^{\left(-\frac{k}{m}\right)t}$$

$$\Rightarrow \quad \left[\frac{\left(\frac{mg}{k}-v(t)\right)}{\left(\frac{mg}{k}\right)}\right] = e^{\left(-\frac{k}{m}\right)t}$$

$$\Rightarrow \quad \frac{mg}{k}-v(t) = \left(\frac{mg}{k}\right)e^{\left(-\frac{k}{m}\right)t}$$

$$\Rightarrow v(t) = \frac{mg}{k}\left[1-e^{\left(-\frac{k}{m}\right)t}\right].$$

l.) Notice that our expression makes sense at the extremes. That is:

i.) At $t = 0$, the $e^{-(k/m)t}$ expression becomes "1" (i.e., $e^0 = 1$) and the right-hand side of the equation becomes ZERO. That is exactly what our *velocity* was supposed to be at $t = 0$.

ii.) At $t = \infty$, the $e^{-(k/m)t}$ expression becomes zero (i.e., the expression $e^{-(k/m)t}$ equals $1/e^{(k/m)t}$, which equals $1/e^{\infty} = 0$ at $t = \infty$) and the right-hand side of the equation becomes mg/k. That, if you remember, was the *terminal velocity* of the body. In short, the extremes seem to fit our expression nicely.

6.) Why do we care? The *Advanced Placement Test* has, on occasion, had questions like: *A body accelerates from rest under the influence of a given <u>velocity dependent force</u>. Determine the body's <u>velocity as a function of time</u> (i.e., what is v(t)).* The functions are not always as stinky as the one we have just examined, but the *approach-for-solving* is similar!

F.) Centripetal Force:

1.) A net force acting on a body produces an acceleration, and an acceleration implies a *change of velocity*. So far, the only accelerations we have dealt with have been those that change the *magnitude* of a body's velocity--that make objects *speed up* or *slow down*. There is another way an acceleration can change a body's motion: it can change the <u>direction</u> of the body's *velocity* vector.

2.) Example--the M.O.B. maneuver:

a.) In days of yore when rock-n-roll was in and '55 Chevys were the *hot rod* of choice, there was a technique the execution of which guaranteed, if not romance, at least close contact between a guy and his date. It was called the infamous M.O.B. maneuver.

<u>The Situation:</u> A boy picks up his date (we will assume the boy is driving). The two like one another. The vehicle is a 1955 Chevy without seat-belts (the norm for 1950's cars). It sports a bench seat (also the norm for 1950's cars). She gets in and sits next to the passenger-side door. He'd like her to sit next to him but is too bashful to ask her to move over. She'd *like* to sit next to him but is too shy to do it on her own.

Solution: the M.O.B. maneuver.

The Maneuver: Traveling at 30 miles per hour, the boy casually approaches an appropriate intersection (see Figure 5.19).

Upon reaching the corner, he makes a hard right turn without warning.

Consequence: she finds herself seated next to him in short order.

How so?

He is attached to the turning car via the steering wheel. As the car turns, the steering wheel exerts a force on him that makes him move on the *same semi-circular path* the car follows. She, on the other hand, is *not* attached to the car (the maneuver works best if the seat has been waxed beforehand). As there are no forces acting on her, she does exactly what any force-free body will do--she continues to move in a *straight line* (Newton's First Law--bodies in motion tend to stay in motion in a straight line unless impinged upon by a force). Doing so insures that he and she will, indeed, meet somewhere around the BINGO in Figure 5.20.

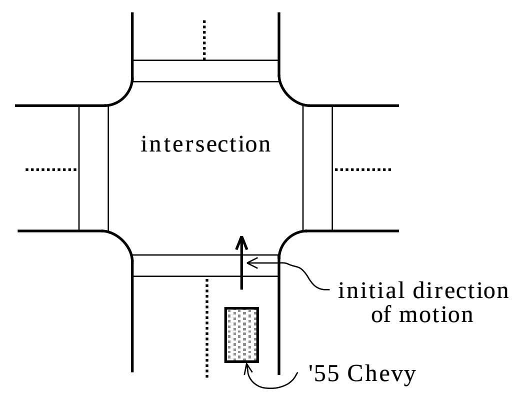

FIGURE 5.19

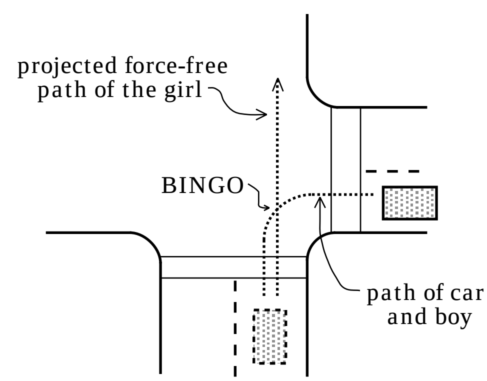

FIGURE 5.20

Note: You might wonder why it is called an MOB maneuver? Although it is probably *politically* and *socially* improper today, in the 1960's it stood for *Move Over, Baby*.

b.) The Point: An acceleration, hence a net force, is needed to *change the direction of a moving body* (hence, the *direction* of the body's *velocity* vector). That force will be perpendicular to the direction-of-motion, which is to say perpendicular to the direction of the velocity vector.

Note 1: The boy entered the curve at 30 mph, and he left the curve at 30 mph. The *magnitude* of his velocity didn't change, but the *direction* did. That would not have been the case if the force provided by the steering wheel had been in any direction other than *perpendicular to the boy's path*.

Note 2: Assuming this direction-changing force is constant in magnitude, it will always make the body move along a circular path about a fixed center. Forces that do this are often referred to as "center-seeking forces."

Note 3: The phrase *center-seeking* is a label only. In a given situation, the combination of forces that collectively qualify for that moniker must exist naturally within the system. They can be *normal* forces, *tension* forces, *gravitational* forces, *frictional* forces, *push-me-pull-you* forces, or some combination thereof.

Note 4: The word "centripetal" means *center-seeking*. It is the *combination of forces that is center-seeking* that is labeled the net *centripetal force*. Again, this is a label only. We are not talking about a new *kind* of force--only a new kind of *situation* in which the same old forces might be applied.

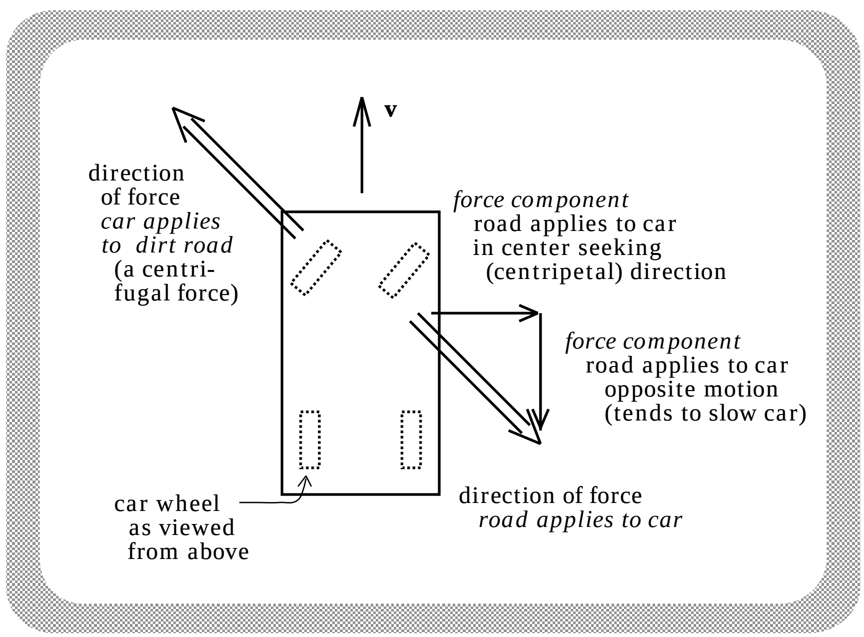

FIGURE 5.21

Note 5: Students often become confused when dealing with *centripetal forces* generated by friction between a car's tires and the road. For clarification, consider the car example above with one modification: assume the car is driving on a *dirt* road. What happens when the car's wheels are cranked into a hard, tight turn (see Figure 5.21)?

The car's tires apply a *frictional force* to the ROAD and end up throwing dirt outward away from the car's ultimate path. The force the *car applies to the road* is, therefore, directed outward relative to the ultimate path of the car. On the other hand, the *road* applies a frictional force f_{car} to the car which, according to Newton's Third Law, is *equal in magnitude* and *opposite in direction* to the force the car applies to the road.

This force, f_{car}, has one component that is oriented opposite the direction of the car's motion and another component that is perpendicular to the direction of motion. The latter component pushes the CAR inward, making it follow a curved path. This is the center-seeking (*centripetal*) force in the system.

When using Newton's Second Law to analyze the motion of the car, the only forces we are interested in are the forces *acting on the car*. As such, the force the car applies to the road has no place in the analysis.

3.) Other Examples:

a.) Consider the moon orbiting the earth: The moon does not move off into space in a straight line because the earth exerts a *gravitational force* on the moon, pulling it away from the path it would have taken if it had been force-free (see Figure 5.22). Consequence: the moon is pulled into a nearly circular path around the earth.

In this case, *gravity* is the *center-seeking* (centripetal) force that naturally exists within the system.

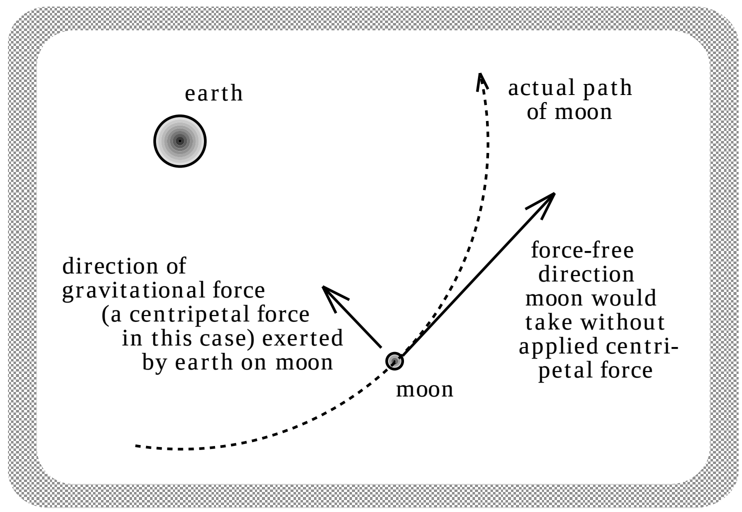

FIGURE 5.22

Note: The moon's path isn't perfectly circular due to the initial conditions under which the moon's motion was generated.

b.) The *hammer throw* in Olympic competition is executed using a mass (a ball) attached to a chain. The contestant grasps the chain, swings it around and around until he or she reaches a relatively high spin-speed, and then lets the ball and chain go.

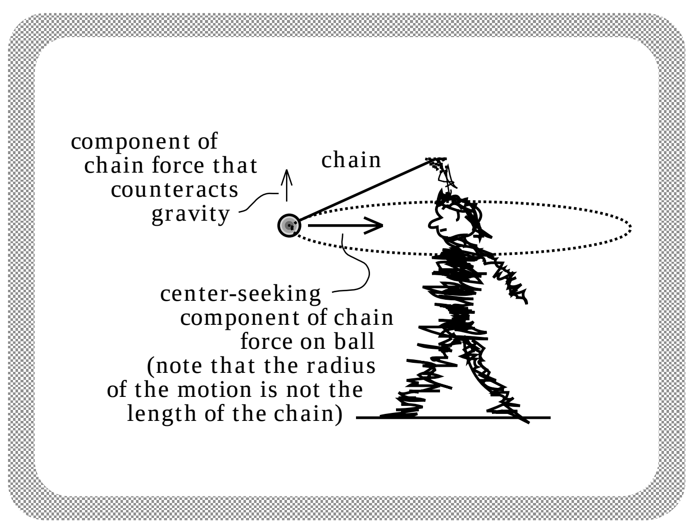

FIGURE 5.23

The proper technique does not suggest twirling the ball overhead in a horizontal plane, but let's assume old Joe is a bit off-form and does not know any better (see Figure 5.23). From whence does the *center-seeking* (centripetal) force on the ball come?

It comes from the *horizontal component* of the *tension* force provided by the chain. That component is the *centripetal force* in the system.

4.) Example Problem--Deadman's Curve: A car approaches a curve of known radius *R*. The *coefficient of static friction* between the tires and the road is μ_s. At what *maximum velocity* can the car take the curve without breaking traction and spinning out (Figure 5.24 shows situation from above)?

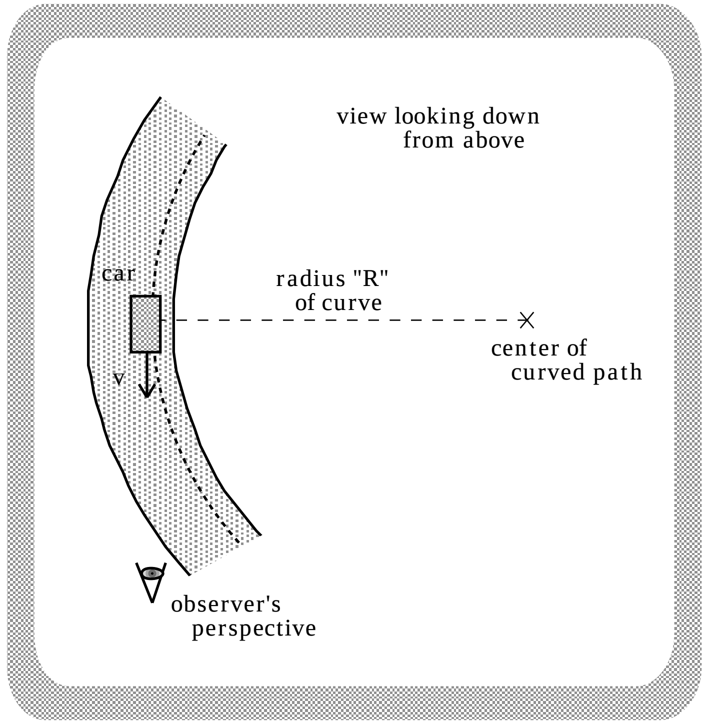

FIGURE 5.24

Note: Assume the wheels are turned so slightly that the component of f_s that opposes the car's motion is zero. That is, assume f_s is fully centripetal.

a.) This is a Newton's Second Law problem. We need to identify the important forces acting on the car, so we will use an f.b.d. perspective that views the car head-on (the view shown in Figure 5.25 depicts that perspective).

b.) For the vehicle to follow a circular path, we know there must be a force, hence acceleration, in the center-seeking direction (i.e., in the direction oriented *toward the center of the circle upon which the car is moving*).

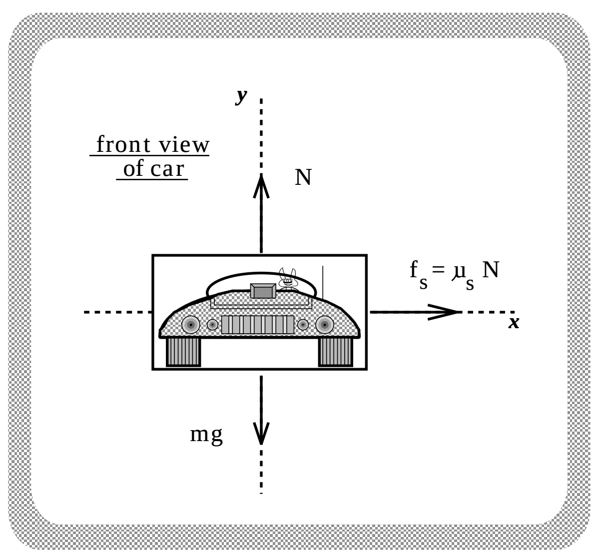

FIGURE 5.25

Note: The *centripetal direction* will be *perpendicular* to the direction of motion. As seen in our sketch, the car is coming *out of the page* which means the center-seeking direction is to the right in the plane of the page.

c.) Using N.S.L., we get:

$\underline{\sum F_y:}$

$$N - mg = ma_y.$$

As $a_y = 0$ (the car is not jumping up off the road), we get:

$$N = mg.$$

$\underline{\sum F_{center\text{-}seeking}:}$

$$\mu_s N = ma_c.$$

Substituting in $N = mg$ from above and solving for a_c, we get

$$\begin{aligned} a_c &= \mu_s(mg)/m \\ &= \mu_s g. \end{aligned}$$

Note: This will be the maximum acceleration the road/tire contact can generate. How so? Because $\mu_s N$ is not just any static frictional force, it is the *maximum* static frictional force. It will be related to the maximum *centripetal acceleration* available to the car when it attempts to follow the curve, which in turn is related to the maximum *velocity* possible.

d.) If we had a constant acceleration whose direction was *tangent to the curve*, the velocity's magnitude would change with time and we could determine $v(t)$ by solving the expression $a = dv/dt$. The problem? The acceleration a_c is *radially* directed. That means it is not oriented to change the velocity's magnitude--it is oriented to change the velocity's direction. As such, neither $a = dv/dt$ nor kinematics will help us.

We need to derive a relationship between the *velocity of the car* and the *acceleration* component that is changing the car's *direction* of motion (i.e., the centripetal acceleration). That derivation is at the end of the chapter (you'll never be asked to reproduce that derivation by yourself--it has been included so that you can convince yourself that nothing *dirty* has been done here).

e.) <u>The bottom line of that derivation:</u> if a body moving with *velocity magnitude v* is to move into a circular path of radius *R*, it must experience a center-seeking (centripetal) acceleration a_c numerically equal to v^2/R.

f.) With this information, we return to our problem. If $a_c = v^2/R$ and, from above,

$$a_c = \mu_s g,$$

we get

$$v^2/R = \mu_s g$$

$$\Rightarrow \quad v = (\mu_s R g)^{1/2}.$$

Note 1: This solution does make sense intuitively. Think about your own experience on the highways and byways of America: tight turns are taken slowly while wide turns are taken at higher speeds. That is exactly what our equation predicts. The larger the curve (i.e., the bigger R), the larger the predicted maximum velocity.

Note 2: The units for the variables under the radical had better work out to *meters per second*. Checking: the coefficient of static friction is unitless; R has the units of *meters*; and g the units of *meters per second squared*. The net units under the radical are *meters squared per seconds squared*, the square root of which is *meters per second*.

5.) Example Problem 2--Deadman's Curve with Banked Incline: Our car now approaches a banked curve of angle θ, radius R, and coefficient of static friction μ_s. What is the maximum velocity v_{max} with which the car can take the curve without flying up over the top of the embankment?

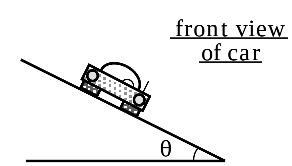

FIGURE 5.26

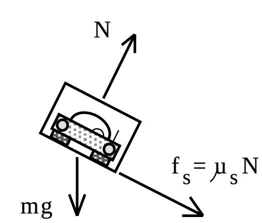

FIGURE 5.27

Note 1: As viewed from above, the car moves in a semi-circular path that looks exactly like the path taken by the car in the previous problem. We will, therefore, use Figure 5.24 to depict the overhead view here. A head-on perspective is shown in Figure 5.26. A *free body diagram* for the situation is shown in Figure 5.27.

Note 2: The *maximum velocity* will occur when the maximum static frictional force just holds the car from breaking loose and sliding up over the top of the bank. That frictional force must, therefore, be directed down the incline.

Note 3: One axis must be along the line of centripetal acceleration. That *center-seeking direction* is oriented along the line of the RADIUS of the car's motion. That direction is *not* along the line of the incline. That is, even though the car is physically tilted, the direction toward the CENTER of the car's circular path is in the *horizontal*. As such, one of the axes has been defined in that direction.

a.) Examining the modified *free body diagram* (modified in the sense that axes and force components have been included) shown in Figure 5.28, Newton's Second Law yields:

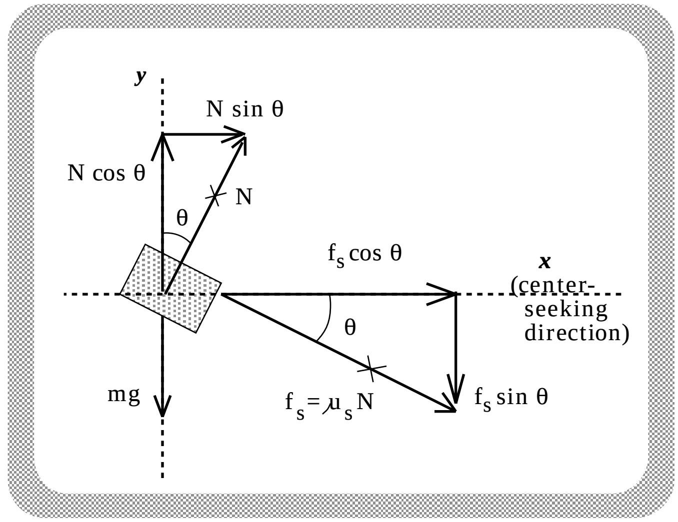

FIGURE 5.28

$\underline{\sum F_y:}$

$$N\cos\theta - mg - f_s\sin\theta = ma_y.$$

b.) As $a_y = 0$, and noting that $f_s = \mu_s N$, we get:

$$N\cos\theta - mg - (\mu_s N)\sin\theta = 0$$
$$\Rightarrow \quad N = (mg)/(\cos\theta - \mu_s\sin\theta).$$

c.) In the *center-seeking* direction:

$\underline{\sum F_{center\text{-}seeking}:}$

$$N\sin\theta + f_s\cos\theta = ma_c.$$

d.) Substituting in $f_s = \mu_s N$, $N = [(mg)/(\cos\theta - \mu_s\sin\theta)]$, and letting $a_c = v^2/R$, the equation

$$N\sin\theta + \mu_s N\cos\theta = ma_c$$

becomes

$$[(mg)/(\cos\theta - \mu_s\sin\theta)]\sin\theta + \mu_s[(mg)/(\cos\theta - \mu_s\sin\theta)]\cos\theta = mv^2/R.$$

e.) Dividing out the *m's* and rearranging, we get:

$$v_{max} = [R\,(g \sin\theta + \mu_s g \cos\theta)/(\cos\theta - \mu_s \sin\theta)]^{1/2}.$$

Note 1: When $\theta = 0$ (i.e., the same situation we had in the Example Problem 1), we get $v_{max} = (\mu_s Rg)^{1/2}$ as expected.

Note 2: For $\theta = 30^o$, $\mu_s = .25$ (this corresponds to a relatively slick roadway), and a radius of 50 meters, the calculated value of $v_{max} = 21.77\ m/s$. This is right around 50 mph.

6.) Alternate Example--Problem 2: The *free body diagram* in Figure 5.29 presents another possibility. If the angle of the inclined road is great enough, it is possible that the maximum static frictional force on a stationary or slowly moving car would not be large enough to hold the car on the incline. Consequence: the car would break loose and slide *down* the incline.

In that case, a car could make it through the turn but it would have to be moving with a velocity above some minimum value. As the car would want to slide down the incline for velocities below that minimum, the static frictional force would be up the incline (the car wants to slide down).

Consider the car and embankment used in *Part 5* above. What is the *minimum velocity* v_{min} the car can take the curve without sliding down into the infield at the bottom of the embankment?

a.) The f.b.d. for this situation is shown in Figure 5.29:

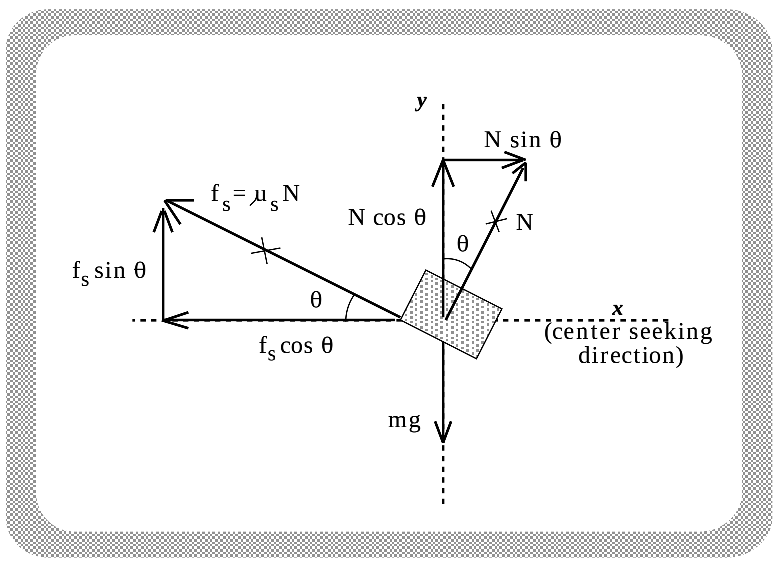

FIGURE 5.29

b.) N.S.L. maintains:

$\underline{\sum F_y:}$

$$N \cos\theta - mg + f_s \sin\theta = ma_y.$$

As $a_y = 0$ and $f_s = \mu_s N$, we get:

$$N \cos\theta - mg + (\mu_s N) \sin\theta = 0$$

$$\Rightarrow \quad N = (mg) / (\cos\theta + \mu_s \sin\theta).$$

c.) In the *center-seeking* direction:

$\underline{\sum F_{\text{center-seeking}}:}$

$$N \sin\theta - f_s \cos\theta = ma_c.$$

d.) Substituting $f_s = \mu_s N$; substituting the quantity $[(mg)/(\cos\theta + m_s \sin\theta)]$ for N; and setting $a_c = v^2/R$, the equation

$$N \sin\theta - \mu_s N \cos\theta = ma_c$$

becomes

$$[(mg)/(\cos\theta + \mu_s \sin\theta)]\sin\theta - \mu_s[(mg)/(\cos\theta + \mu_s \sin\theta)]\cos\theta = mv^2/R.$$

e.) Dividing out the *m's* and rearranging, we get:

$$v_{min} = [R\,(g \sin\theta - \mu_s g \cos\theta)/(\cos\theta + \mu_s \sin\theta)]^{1/2}.$$

Note 1: When $\theta = 0$ (the situation we had in the Example Problem 1), we get:

$$v_{min} = (-\mu_s Rg)^{1/2},$$

an imaginary value. This implies that there is *no minimum speed* at which a car could take a flat curve and *not* make it through.

Note 2: For $\theta = 30^o$, $\mu_s = .25$ (again, this is a very slick roadway), and a radius of 50 meters, the calculated value of $v_{min} = 11.9\ m/s$. This is around 25 mph. Below this speed, the car will not hold traction with the road and will slide down toward the bottom of the bank.

7.) <u>Derivation of Centripetal Acceleration Expression</u>: You will not be asked to duplicate this proof; it is included for the sake of completeness.

It was stated above that the magnitude of any *centripetal acceleration* acting on an object moving in a circular path is numerically equal to v^2/R, where v is the magnitude of the object's velocity and R the radius of the path. The derivation from which that claim comes follows:

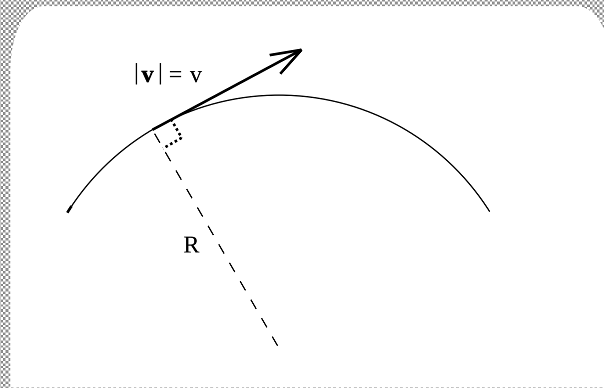

FIGURE 5.30a

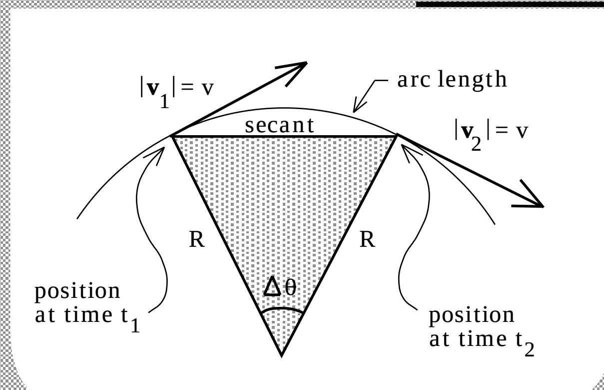

FIGURE 5.30b

a.) A body with a constant velocity-magnitude v moves on a circular path of radius R as shown in Figure 5.30a. Between times t_1 and t_2, the body moves through an angular displacement of $\Delta\theta$. Figure 5.30b shows this set-up along with a secant connecting the position points at times t_1 and t_2. Notice that:

i.) The triangle is isosceles (two of the sides are equal in length to the radius R of the motion);

ii.) The third side of the triangle (the secant) is identified as Δs;

iii.) Δs and the arc length subtended by $\Delta\theta$ are not equal as long as $\Delta\theta$ is large; and

iv.) The arc length is numerically equal to the velocity-magnitude v times the time interval (Δt) required to travel between positions 1 and 2, or:

$$\text{arc length} = v\ \Delta t.$$

b.) The vector difference between the velocity vectors at t_1 and t_2 yields a change of velocity vector $\Delta \mathbf{v}$ over that period of time. That vector subtraction is shown in Figure 5.31. Notice that:

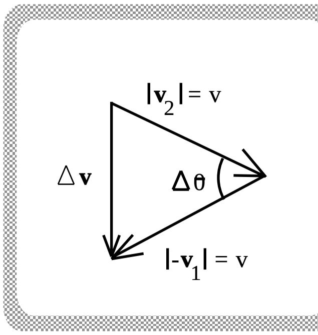

FIGURE 5.31

i.) The direction of the vector difference Δv is approximately oriented toward the center of the circular path on which the body moves;

ii.) The triangle created by the subtraction is isosceles (two of the sides are numerically equal to the magnitude of the constant velocity v); and

iii.) Because the velocity vectors are perpendicular to their respective radii, the isosceles triangle created by the radii and secant (Figure 5.30b) is *similar* to the triangle created by the velocity-vector-difference (Figure 5.31).

c.) With *similar* triangles, the ratio of any two sides of one triangle will be equal to the ratio of the corresponding sides of the second triangle. As such, we can write:

$$\Delta v/v = \Delta s/R.$$

d.) If the time interval is allowed to become very small (i.e., t_1 and t_2 approach one another), the arc length approaches Δs and we can write:

$$\text{arc length} = \Delta s \text{ (in the limit as } \Delta t \text{ goes to zero).}$$

But the arc length equals $v\Delta t$, which implies that as $\Delta t \to 0$,

$$\Delta s \to v\Delta t.$$

e.) Plugging this back into our ratio, we can re-write $\Delta v/v = \Delta s/R$ as:

$$\lim_{\Delta t \to 0}\left(\Delta v / v\right) = \left(v\Delta t / R\right).$$

Rearranging yields:

$$\lim_{\Delta t \to 0}\left(\Delta v / \Delta t\right) = \left(v^2 / R\right).$$

f.) The left-hand side of this expression is the definition of instantaneous acceleration. What acceleration? The acceleration whose direction is *center-seeking*.

Bottom line: The *magnitude* of the *centripetal acceleration* required to move a body whose velocity magnitude is v into a circular path of radius R is v^2/R.

G.) A Note About MASS:

1.) Although the material you are about to read will not be included on a test, it is important that you understand what *mass* really is. READ THIS SECTION ONCE. If nothing else, it will help you stave off massive confusion later on when we get to rotational motion.

2.) There are certain characteristics that are true of all material objects. For instance, all objects have *a tendency to resist changes in their motion.*

a.) Example: A rock placed in space will not suddenly, spontaneously accelerate for no reason. It will sit in its place until a force makes it move.

b.) The unwillingness of an object to spontaneously change its motion is called *inertia.* As the amount of inertia an object has is intimately related to how much *force* will be required to *accelerate* the object, quantifying the idea of inertia is important. Early scientists satisfied that need by defining an *inertia-related quantity* they called "inertial mass."

c.) The system they devised is simple. A platinum-iridium alloy cylinder, currently housed in a vault at the *Bureau of Weights and Measures* in Sevres near Paris, France, was defined as having *one kilogram* of inertial mass. All other inertial mass values are measured relative to that cylinder. That is, an object with the same amount of *resistance to changing its motion* as does the standard is said to have "one kilogram of *inertial mass*." An object with twice the *resistance to changing its motion* is said to have two kilograms of *inertial mass*; one-half the resistance implies one-half kilogram of *inertial mass,* etc.

In other words, the inertial mass of a body gives us a numerical way of defining how much inertia an object has RELATIVE TO THE STANDARD. Put still another way, *inertial mass is a <u>relative measure</u> of a body's tendency to resist changes in its motion.*

d.) Although France is a beautiful country, it would be terribly inconvenient for laboratory scientists around the world if they had to travel to France every time they wanted to determine an *inertial mass* value, so scientists further generated a laboratory technique for measuring *inertial masses*. It utilizes an inertial balance--a tray mounted on two thin blades that allow the tray to vibrate back and forth. The more mass that is placed in the tray, the slower the tray vibrates. A simple formula relates the tray's vibratory rate (its *period* of motion) to the amount of *inertial mass* there is in the tray.

Although it works, using an inertial balance is a VERY CUMBERSOME and time consuming operation.

3.) There are other characteristics that are true of material objects. For instance, all massive objects are attracted to all other massive objects (at least according to Newton). We call this attraction *gravity*.

a.) A measure of a body's *willingness to be attracted to another body* is related to what is called the "gravitational mass" of an object.

b.) To provide a quantitative measure of *gravitational mass*, scientists have taken an agreed upon object as the standard against which all subsequent *gravitational mass measurements* are made (again, this standard is housed today in Sevres, France).

c.) The technique for measuring *gravitational mass* utilizes a *balance* or *electronic scale*. The object is placed on a scale which consists of a spring-mounted pan. The gravitational attraction between *the object and the earth* pulls the object toward the earth and compresses the spring in the process. The scale is calibrated to translate spring-compression into gravitational mass (assuming that is what the scale is calibrated to read--in some cases, such scales are calibrated to read force, hence American bathroom scales measure in pounds).

MEASURING GRAVITATIONAL MASS IS EASY.

4.) Somewhere down the line someone noticed a wholly unexpected and profoundly improbable relationship between gravitational and inertial mass. It was observed that if the same standard (that is, the same object) was used for both, a second object with twice the *gravitational mass* relative to the standard would also have twice the *inertial mass*.

a.) THIS DOES NOT HAVE TO BE THE CASE. There is no obvious reason why a body with twice the *resistance to changing its motion* (relative to the standard) should also have twice the *willingness to be attracted to other objects*. The two characteristics are completely independent of one another, yet they appear to parallel one another to a high degree of precision (in fact, the best comparisons to date have accuracy to around 10^9 with *no discrepancy found* even at that order of magnitude).

b.) Scientists could have called the units of *gravitational mass* anything they wanted, but because they knew the parallel between *gravitational mass* and *inertial mass* existed, they decided to give *gravitational mass* the units of "kilograms."

That means that as defined, a body with two kilograms of *gravitational mass* also has two kilograms of *inertial mass*.

c.) The beauty of this choice is obvious. Newton's Second Law relates a body's acceleration to the force applied to it. The proportionality constant--the mass term--is an *inertial mass* quantity (the body's resistance to changing its motion is the characteristic that governs how much acceleration the body will feel when a given force is applied). Determining *inertial masses* in the lab is a pain in the arse--inertial balances are not easy devices to set up or use. But *gravitational mass* is easy to measure. All you need is an electronic balance. So when you or I or any lab technician needs to know a body's *inertial mass*, all we have to do is measure the body's gravitational mass on a scale and we have what we want.

d.) <u>Bottom line</u>: Due to the parallel, most people no longer distinguish between *gravitational mass* and *inertial mass*. As the two are numerically interchangeable, people nowadays simply refer to a body's "mass" and leave it at that.

5.) There is an interesting consequence of this peculiar parallel between *gravitational mass* and *inertial mass* which is most easily observed by considering the following question: If a body with two kilograms of *gravitational mass* is attracted to the earth twice as much as a body with only one kilogram of gravitational mass, why doesn't the two kilogram mass free fall toward the earth faster than the one kilogram mass?

a.) The answer is simple. A body with twice the *gravitational mass* also has twice the *inertial mass*; it needs a greater force to overcome its greater inertia.

b.) In short, it does not matter how massive an object is, its *inertia* and its *willingness to be attracted to the earth* will always balance one another out making the object accelerate at the same rate as all other objects (assuming you ignore air friction, etc.).

H.) Fictitious Forces: Centrifugal And Others:

1.) Although the phrases look and sound similar, the ideas behind *centripetal force* and *centrifugal force* are patently different.

a.) The phrase *centripetal force* is a label used to identify forces that motivate a body out of straight line motion and into curved motion. These *center-seeking forces* must exist *naturally* within a system.

b.) The phrase *centrifugal force* is a label used to identify forces that must be ASSUMED TO EXIST if one is to use Newton's Second Law in analyzing problems from a non-inertial (i.e., accelerated) frame of reference. What does this mean? Read on.

2.) Consider the following example (a slight modification of the M.O.B. maneuver): You are driving in your car. You notice that the box sitting next to you slides *away from you* as you make a left turn (Figure 5.32). Does that mean there is a force acting to push the parcel toward the right door?

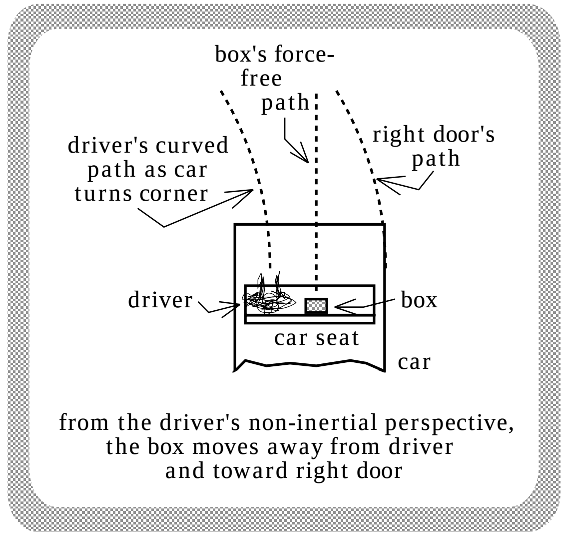

FIGURE 5.32

a.) The answer to the question is *no*. The only forces that are acting on the parcel are gravity and a normal force from the seat (we'll ignore friction). So what is going on?

b.) What is really happening is that *you* are moving away from the parcel (it continues moving in a straight line path, relative to the street) as you and the car are centripetally forced into the turn.

c.) The problem? If you wanted to analyze the parcel's motion from *your frame of reference* (i.e., from a frame attached to the centripetally accelerating car), Newton's Second Law shouldn't work. Why? Because N.S.L. only works in non-accelerated frames of reference.

d.) What's nice is that you *can* make N.S.L. work if you are clever. If you assume there exists a *fictitious force* acting to push the parcel away from you, Newton's Laws will work just fine. For circular motion, that force is called a *centrifugal force*. Its magnitude is equal to mv^2/R. In short, you don't *have* to analyze the problem from a non-inertial frame of reference, but you can if you are so disposed.

3.) After all that, reconsider the initial statement in *Part H-1a* and *b*:

a.) The phrase *centripetal force* is a label used to identify forces that motivate a body out of straight line motion and into curved motion. These *center-seeking forces* must exist *naturally* within a system.

b.) The phrase *centrifugal force* is a label used to identify forces that must be ASSUMED TO EXIST if one is to use Newton's Second Law in analyzing curved-path problems from a non-inertial (i.e., accelerated) frame of reference.

Big Note: What you are about to see is *for your own edification only*. You will not be tested on it. It has been included because the idea of fictitious forces is used a lot at the college level in analysing certain types of physics problems. If you ever need to understand the approach, the following should help.

4.) You are tethered to the center of a rotating platform (Figure 5.33a). You feel a frictional force that keeps you from sliding laterally across the platform, you feel gravity, you feel a normal force, you feel the force of the tether pulling you in, and you feel what appears to be a force pushing you outward. Let's asume you've measured the tension in the tether and found it to be T. How fast is the platform moving? Analyze this situation from both an inertial and a non-inertial frame of reference.

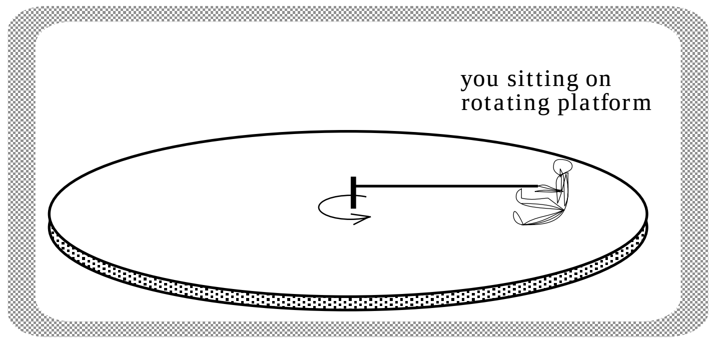

FIGURE 5.33a

a.) From an inertial frame of reference (i.e., a *stationary* observer looking from the platform's side), the forces acting on you are shown in Figure 5.33b. Using N.S.L., we can write:

$\underline{\sum F_x:}$

$$T = m(a_x)$$
$$= m(v^2/R)$$
$$\Rightarrow \quad v = (TR/m)^{1/2}.$$

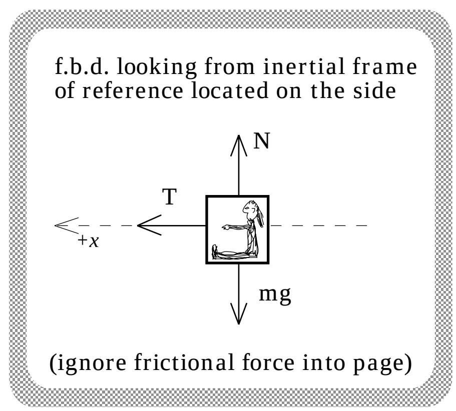

FIGURE 5.33b

b.) From a non-inertial frame of reference (i.e., someone sitting next to you on the platform), the forces acting on you are shown in Figure 5.33c. Noting that as far as you are concerned, you aren't accelerating at all (you don't seem to be moving along the line of the tether), we can use N.S.L. to write:

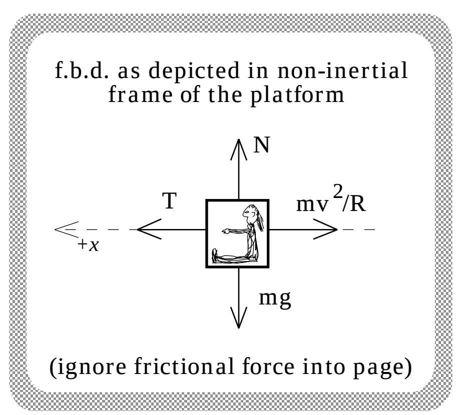

FIGURE 5.33c

$\underline{\sum F_x:}$

$$T - m(v^2/R) = ma_x = 0 \qquad \text{(as } a_x = 0 \text{ in this frame)}$$

$$\Rightarrow \quad v = (TR/m)^{1/2}.$$

c.) The velocities determined in both frames are the same.

Note: The trick is to make the actual *ma* value, as observed in the inertial frame (this would be mv^2/R in this case), into a fictitious force in the non-inertial frame, then to look to see if there appears to be an apparent acceleration in the non-inertial frame. In the *car rounding the corner* example, the box appeared to be accelerating away from you, so in that case, there is an acceleration as viewed in the non-inertial frame of your car. In the above example, there doesn't appear to be acceleration in the non-inertial frame as you move around the circle, so $a = 0$ in that case. In both cases, though, you took the real, inertial acceleration (for these cases, v^2/R), multiplied it by m, and treated it as though it were a force.

5.) One more *fictitious force* example: A mass on a spring is placed in an accelerating rocket (Figure 5.34a) out in space (i.e., no gravity). The rocket's acceleration is known to be *A*. Assuming you also know the spring constant, by how much is the spring elongated from its equilibrium position (the variable *y* in the sketch)? Analyze the situation from both the inertial frame of someone watching from outside the rocket, and then from the non-inertial frame of someone sitting inside the rocket with the spring.

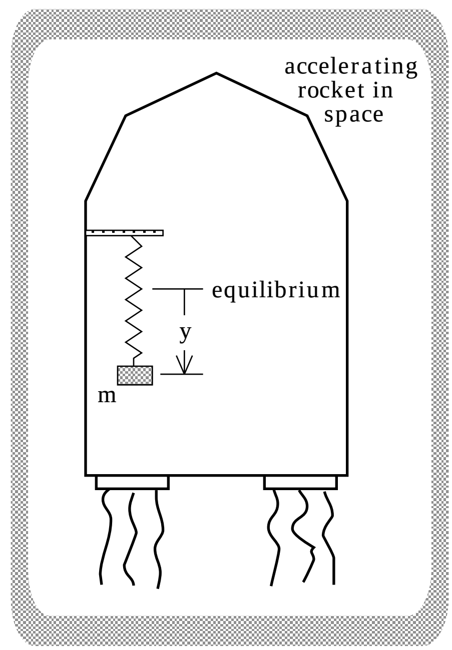

FIGURE 5.34a

a.) From an inertial frame of reference (i.e., a stationary observer outside the rocket), the only force accelerating *m* in this gravity free situation is *ky* (Figure 5.34b). Using N.S.L., we can write:

$\underline{\sum F_y:}$

$$ky = m(a_y)$$

$$= m(A)$$

$$\Rightarrow \quad y = (mA/k).$$

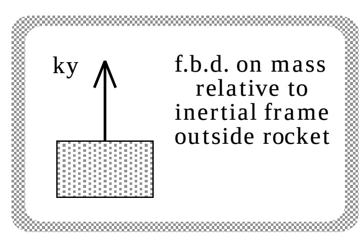

FIGURE 5.34b

b.) From a non-inertial frame of reference (i.e., inside the rocket), there are two apparent forces acting: *ky* pulling up and a fictitious force equal to *mA* pulling the spring down (Figure 5.34c). Noting that *m* is not accelerating relative to the rocket, N.S.L. yields:

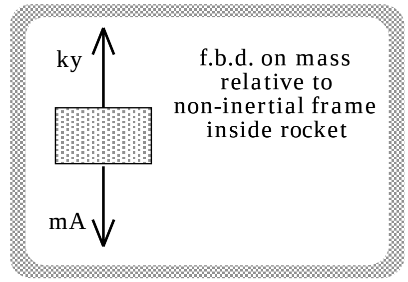

FIGURE 5.34c

$\underline{\sum F_y:}$

$$ky - m(A) = 0$$ (as $a = 0$ as viewed from inside the frame)

$$\Rightarrow \quad y = (mA/k).$$

c.) Huzzah! The *y* values calculated in both cases are the same.

d.) Now re-read the first sentence of the Note just above H-5.

I.) Determining A Time Dependent Velocity *v(t)* From N.S.L.:

1.) There is one class of N.S.L. problems in which unembedding acceleration signs can get you into serious trouble. As was the case in the *Friction and Free Fall* section (*Section E*), it is the situation in which a body's *velocity as a function of time--v(t)*--is being sought.

2.) Though it may not have been obvious at the time when we did the frictional free fall problem:

a.) We identified the frictional force as having a magnitude of *kv*, where *v* was the *magnitude* of the body's velocity vector (remember, we had to manually insert a positive or negative sign when writing out the N.S.L. expression for the situation).

b.) N.S.L. was used to relate the force quantity *kv* to the acceleration *dv/dt* yielding a differential expression we could solve.

c.) Though it was, again, probably not obvious, the *dv/dt* notation alluded to above *had* to be defined as the derivative of the velocity *MAGNITUDE*. Otherwise, the *v* and *dv/dt* terms would have been the mathematical equivalent of apples and oranges, one being a vector and the other a magnitude.

d.) Because the problem was set up as it was, we got away with being notationally sloppy without disastrous consequences. You are about to see what those consequences could have been if we hadn't been so lucky, and how you can avert the problem altogether.

3.) To understand the difficulty, consider the following:

a.) A mass moves *to the right* on a frictionless surface. A force is applied to the mass *to the right* (see Figure 5.35). During the body's motion, the magnitude of the body's velocity *increases* and the *velocity's magnitude-change* is *positive.*

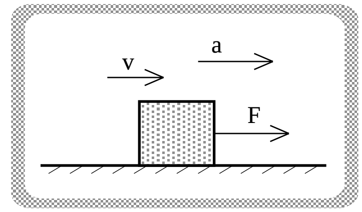

FIGURE 5.35

i.) Justification? If the body's velocity increases from 5 m/s to 7 m/s, the change of the velocity magnitude will be $\Delta v = (7\ m/s - 5\ m/s)$ . . . a positive quantity.

ii.) What about *dv/dt*? In this case, *dv/dt* is inherently positive (that is, just as Δv has a positive sign embedded within itself, so does *dv* and, hence, *dv/dt*).

iii.) So what does N.S.L. do for us? Summing the forces in the *x direction* yields:

$$F = m(dv/dt).$$

iv.) Kindly notice that this expression makes perfect sense as it stands. The force is inherently positive, *dv/dt* is inherently positive, the two sides of the equation are equivalent and all is well.

b.) Now, consider the same mass moving *to the left* on the same frictionless surface. The force is still being applied *to the right* (see Figure 5.36). During the body's motion, the magnitude of the body's velocity *decreases* and the *velocity's magnitude-change* is *negative.*

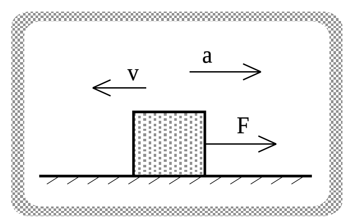

FIGURE 5.36

i.) Justification? If the body's velocity drops from 5 m/s to 3 m/s, the change of the velocity *magnitude* will be $\Delta v = (3\ m/s - 5\ m/s)$ . . . a negative quantity.

ii.) What about *dv/dt*? In this case, *dv/dt* is inherently negative (that is, just as Δv has a negative sign embedded within itself, so does *dv* and, hence, *dv/dt*).

iii.) So what does N.S.L. do for us? As the freebody diagram hasn't changed (velocities are not included on them), summing the forces in the *x direction* yields:

$$F = m(dv/dt).$$

iv.) Oops. We have a problem. The force is still inherently positive, but *dv/dt* is now inherently negative. The only way the two sides of the equation can equal one another is if a negative sign is inserted in front of the *ma* term. This insertion would NOT be the consequence of the acceleration (as a vector) being negative. In fact, if you take the derivative of the velocity *vector*, you will get a positive acceleration. It is the consequence of the fact that the derivative we are taking is that of the velocity *magnitude*. That derivative is inherently negative. In short, the N.S.L. expression that will allow us to derive a reasonable velocity function (i.e., one that sees the velocity slowing with time) is:

$$F = - m(dv/dt)$$

c.) Summarizing, in both cases the acceleration is *to the right*, yet depending upon the direction of motion, the sign in front of the *m(dv/dt)* term can either be positive or negative.

5.) So how *do* we remedy the difficulty when dealing with N.S.L. problems in which we are asked to determine a body's velocity as a function of time?

a.) The easiest way to do this is also the most mathematically satisfying. Simply:

i.) Write out the velocity vector, sign unembedded.

ii.) Take the time derivative of that velocity function. That will give you the appropriate $\pm dv/dt$ term.

b.) Example: Consider the block moving *to the left* (i.e., in the *-**i*** direction).

i.) The velocity vector in that case can be written as:

$$\begin{aligned}\mathbf{v} &= v\,(-\mathbf{i}) \\ &= -v\,(\mathbf{i}),\end{aligned}$$

where *v* is the magnitude of the velocity vector at any arbitrary point in time and the unit vector ***i*** is directed along the line of motion (note that the negative sign has been detached from the unit vector).

ii.) Taking the time derivative yields:

$$\begin{aligned}\mathbf{a} &= \left(\frac{d\mathbf{v}}{dt}\right)\\ &= \frac{d}{dt}\left[-v(\mathbf{i})\right]\\ &= \left(-\frac{dv}{dt}\right)\mathbf{i}.\end{aligned}$$

iii.) N.S.L. becomes:

$$\begin{aligned}F &= ma\\ &= m\left(-\frac{dv}{dt}\right)\\ &= -\ m\left(\frac{dv}{dt}\right).\end{aligned}$$

It works!

c.) <u>Bottom Line:</u> Whenever you find yourself in a situation in which you must use N.S.L. to determine the magnitude of a body's velocity as a function of time, the acceleration term to be substituted into the right side of N.S.L. should be determined using the approach outlined in *Part 5a*.

6.) <u>Parting Shot</u>: Going back to the free fall problem discussed in *Section E*, the positive direction was defined as up, which means the velocity vector should have been written as $\boldsymbol{v} = v(-\boldsymbol{j}) = -\ v(\boldsymbol{j})$. The derivative of this yields an acceleration term of $a = -\ dv/dt$, just as we assumed. The only difference is that now you know *why* the negative sign in front of the acceleration term was appropriate.

QUESTIONS

5.1) Draw a *free body diagram* for the forces acting on each of the bodies in each of the four independent sketches shown in Figure I. Label as completely as you can. <u>Note that in *Part d*, you are being asked to draw f.b.d.s on pulleys. This is for *practice in force recognition*, not because it is something you will ever need to do again</u>. Also, note that in that part, the pin supporting each pulley provides a force.

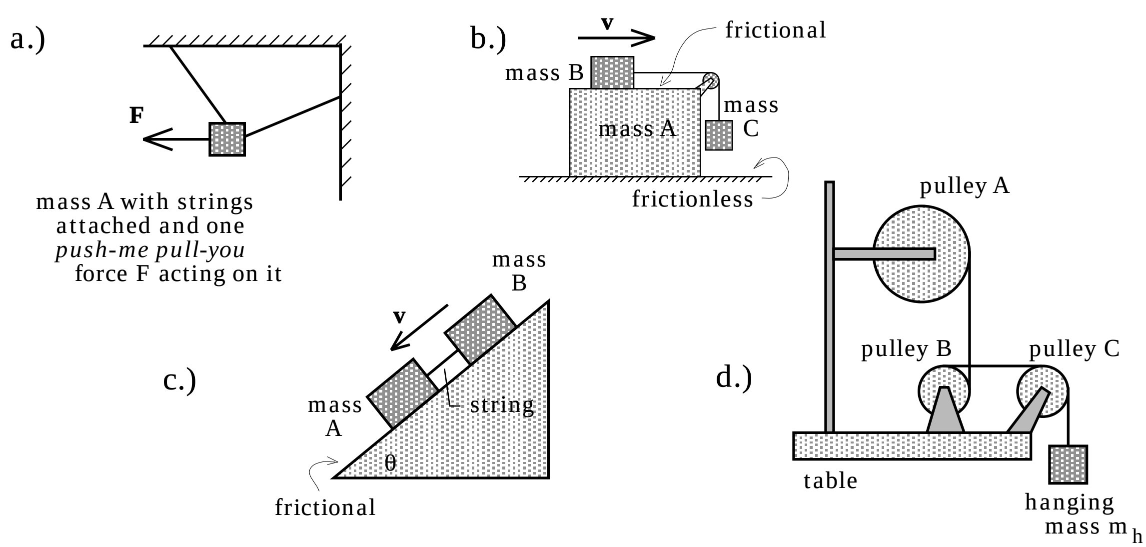

FIGURE I

5.2) In each case below, state the "reaction force" that must exist according to Newton's Third Law.

a.) The floor applies a force to you;
b.) The string applies a force to the weight;
c.) The car applies a force to the tree;
d.) The earth applies a force to the moon.

Important Note: Getting numerical answers for the problems below is not particularly important. You have been given numbers to work with because numbers make students feel secure, but you should initially do each problem algebraically before sticking the numbers in (that is the way you will find the Solutions presented). WHAT IS IMPORTANT is that you can *follow the process* needed to approach and solve these problems, not the bottom line.

BE AWARE: I could very well give you test problems in which there are no numbers, only algebraic symbols.

5.3) A sled whose mass is $m = 30\ kg$ slides horizontally with velocity magnitude $v = 2\ m/s$. It runs into *sticky snow* and experiences a constant *frictional force* of $f_k = 12\ nts.$

a.) What is the sled's acceleration in the sticky snow? (Note that Figure II is for *Part b* only--there is no force F acting in *Part a*).

b.) Your friend decides to help you by pulling the sled with a force $F = 60\ newtons$ at $\theta = 40^o$ with the horizontal (Figure II). For this case, what is the acceleration of the sled? (Note that the vertical component of F will lighten the load, so to speak, lessening N and the *N-related* frictional force--you'll need to determine the coefficient of friction using the information available in *Part a*, then go from there).

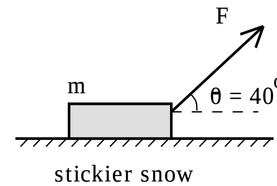

FIGURE II

c.) (**Note:** This will be a difficult question if you are uncomfortable with Calculus; if that be the case, go directly to the Solutions for the approach and execution.) While in the sticky snow, your friend begins to experiment with the *angle of pull* (he has decided that a 40^o pull is not optimal). At what *angle* ϕ will the *minimum force* F_{min} be required to pull the sled *with constant velocity*?

Hint 1: Further explanation: At a large angle, the component of $\boldsymbol{F}$ in the *normal direction* will diminish the normal force $\boldsymbol{N}$ which, in turn, will diminish the frictional force $\mu_k \boldsymbol{N}$. Unfortunately, it will also make the component of $\boldsymbol{F}$ along the line of motion very small--possibly too small to overcome friction. On the other hand, making the angle small will provide more force to counteract friction, but it will do little to diminish the normal force $\boldsymbol{N}$ and, hence, the frictional force. There is an angle that will both diminish $\boldsymbol{N}$ while additionally providing a fairly large component of $\boldsymbol{F}$ in the direction of motion. At that angle, the force required to pull the sled at a constant velocity will be a minimum. Your thrill is to find that angle.

Hint 2: If you can determine the force $\boldsymbol{F}$ as a function of ϕ, you can determine the slope of that function (i.e., $dF(\phi)/d\phi$), then set it equal to zero (see Figure IIa). At the angle at which that is true (i.e., $dF(\phi)/d\phi = 0$), the force will be a minimum.

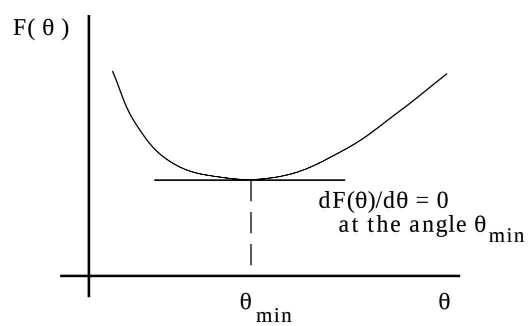

FIGURE IIa

5.4) An elevator whose mass is $m = 400$ *kilograms* is supported by a cable. A frictional force of $f = 80$ *newtons* is applied to the elevator as it moves. Determine the tension T in the cable when the elevator is:

a.) Stationary;

b.) Moving upward while accelerating at 2.8 m/s^2;

c.) Moving upward with a constant velocity-magnitude of 5 m/s;

d.) Accelerating downward at 2.8 m/s^2 with velocity downward;

e.) Moving downward with a constant velocity-magnitude of 5 m/s.

5.5) You are standing on a bathroom scale in an elevator (yes, this is an odd place to find a bathroom scale). Your mass is $m = 60$ *kilograms*; the scale reads $W = 860$ *newtons*. What is the acceleration of the elevator?

5.6) You are sitting in a commercial jetliner. It is on the runway and ready to take off. Before it does, clever soul that you are, you hang a .5 meter long string from the ceiling just overhead and attach to its free end a weight whose mass is $m = .05$ *kg*. The jet begins its acceleration. As it does, the string and mass swing toward you until the string comes into equilibrium at a constant angle of $\theta = 26^o$ with the vertical (see Figure III).

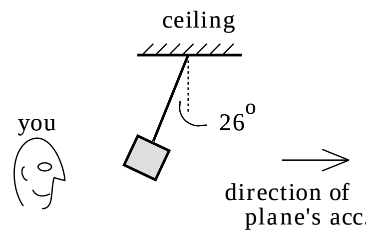

FIGURE III

a.) What is the jet's acceleration?

b.) Once the jet gains altitude and proceeds with a constant velocity, what will the string and mass be doing (that is, will the 26^o angle have changed)? Explain.

5.7) IMPORTANT PROBLEM: Two blocks of mass $m_1 = 2$ *kgs* and $m_2 = 7$ *kgs* are wedged up against one another and against a wall by a horizontal force F (see Figure IV to the right). Doing each section algebraically before putting in the numbers:

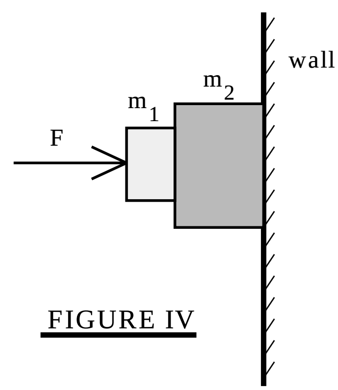

FIGURE IV

a.) What is the *coefficient of static friction* between m_1 and m_2 (call this $\mu_{s,1}$) and between m_2 and *the wall* (call this $\mu_{s,2}$) if the MINIMUM FORCE F_{min} required to keep the blocks from breaking loose and sliding under the influence of gravity if $F_{min} = 25$ *newtons?*

b.) The force F is decreased to 20 newtons. The blocks break loose and begin to fall. If the *coefficients of* <u>*kinetic*</u> *friction* between m_1 and m_2 AND between m_2 and *the wall* are $\mu_{k,1} = .15$ and $\mu_{k,2} = .9$ respectively, what are the accelerations of m_1 and m_2? Note that they will NOT be the same.

c.) Why are the accelerations different?

5.8) IMPORTANT PROBLEM: Known masses $m_1 > m_2$ are attached by a string while sitting on an incline of known angle θ (see Figure V). If the coefficient of friction is a known μ_k and if the two are initially sliding UP the incline (no, you don't know *how* they managed it--somebody obviously gave them a push somewhere along the line--what matters is that they are moving UP the incline *when you see them*):

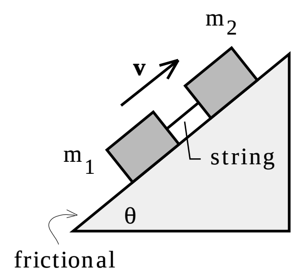

FIGURE V

a.) Draw an f.b.d. for the forces acting on m_1 and m_2.

b.) Derive an expression for the *acceleration* of the system (that is, the acceleration of either m_1 or m_2--both will be the same) as shown.

c.) Derive an expression for the *tension* in the string as shown.

5.9) IMPORTANT PROBLEM: A crate whose mass is m_1 is placed on an incline whose angle is θ. The *coefficient of friction* between the crate and the incline is a known μ_k. A wire attached to the mass proceeds over a frictionless, massless pulley and is attached to a hanging mass m_2. At the instant pictured in Figure VI, m_1 is moving *down* the incline. Additionally, at the instant shown the wire is at an angle ϕ with the line-of-the-incline. *For the moment shown* in the figure, derive general algebraic expressions from which you could determine the crate's acceleration. <u>DO NOT SOLVE THESE EQUATIONS</u>, just generate them!

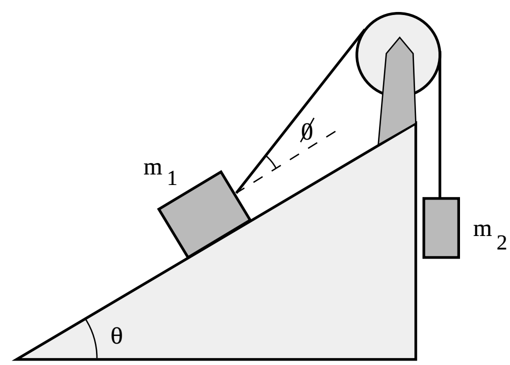

FIGURE VI

Hint: This is not as hard as it looks. <u>Do not look ahead</u>! Content yourself with *following the steps* you have been given in an orderly fashion. This problem will fall out nicely if you do it in pieces. (See additional note below!)

NOTE: This problem is designed to make you think about NSL, but it has one unfortunate characteristic that is not typical of all NSL problems. The acceleration of m_2 is in the same direction and has the same magnitude as the acceleration of the *string* attached to it (this is called a_{string} in the sketch). Unfortunately, the string is attached to m_1 *at an angle*, so m_1*'s* acceleration and the string's acceleration aren't the same. If we call m_1*'s* acceleration a_1 and m_2*'s* acceleration a_2, the relationship between the two parameter's is $a_1 = a_2 \cos\,\phi$. I'm pointing this out because it is very much a side issue. I don't want you spending a lot of time stewing over it.

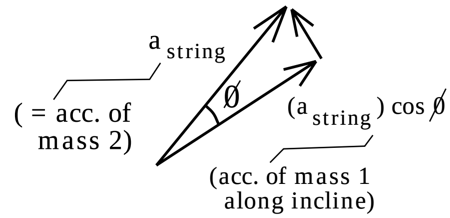

FIGURE VI, supplemental

5.10) A mass m_1 sitting on a frictionless table is attached to a string. The free end of the string is threaded through a hole in the table. Once through, it is attached to a hanging mass m_2 (see Figure VII). If m_1 is then given a shove that moves it into circular motion, what must its velocity be if its radius is to be *R*?

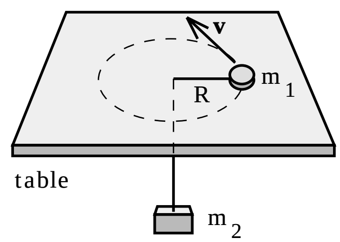

FIGURE VII

5.11) The Loop is a favorite Magic Mountain roller coaster ride that has within it a complete, vertical, 360^o loop. The carts that travel through the loop are attached to the track; let's assume one comes loose. What is the *minimum velocity* magnitude required for such a cart to make it through the top of the loop without coming off the track?

Note: At minimum velocity, the cart will just barely free fall through the top of the arc. In that case, the *normal force* applied to the cart by the track will be zero.

5.12) A particular brand of string can withstand a force of $T = 50$ *newtons* of tension before breaking. A rock of mass $m = .2$ *kg* is attached to a string of length $L = 1.2$ *meters*. A kid then takes the *string and mass* and whirls them around her head (see Figure VIII on the next page).

a.) Assuming the radius of motion is, to a good approximation, equal to the string's length (that is, assume the mass is moving very fast and the *hang-down* angle is zero), what is the *maximum speed* at which the rock can rotate before the string breaks?

b.) Taking the hang-down into consideration (i.e., assume the string's length is no longer equal to the radius of the rock's motion), what is the angle at which the string breaks?

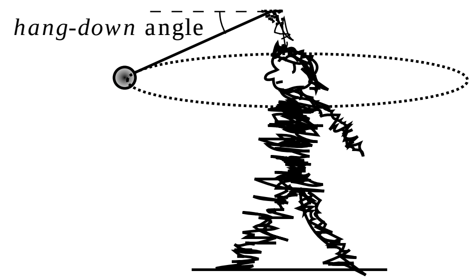

FIGURE VIII

c.) Use the angle determined in *Part b* to determine the velocity of the mass in that situation.

d.) The rock is observed to be hanging at a 30^o angle. How fast is it moving?

5.13) Until now, we have dealt only with gravitational forces between the earth and small objects close to the earth's surface. Under this circumstance, we have taken the *acceleration of gravity* to be nearly constant and equal to $g = 9.8\ m/s^2$. Newton realized that gravity must exist between *any* two objects no matter how far apart they were. To accommodate the most general situation, he derived a general *gravitational force* expression the magnitude of which is:

$$F_g = G\ m_1 m_2/r^2.$$

In this equation, G is called the *Universal Gravitational Constant* and is equal to $6.67\text{x}10^{-11}\ m^3/kg{\cdot}s^2$; m_1 and m_2 are the masses generating the force field, and r is the distance between the *center of mass* of the two objects.

a.) The earth has a *mass* $m_e = 5.98\text{x}10^{24}\ kg$ and a *radius* $r_e = 6.37\text{x}10^6$ *meters*. Assuming *your* mass is 70 kg and you are standing on the earth's surface, what is the force of attraction between you and the earth according to Newton's general expression?

After finishing this calculation, determine the gravitational force between you and the earth, given the assumption that your weight equals mg. How do the two numbers compare?

b.) The *acceleration of gravity* on the earth's surface at the poles is $9.83\ m/s^2$. What is the *acceleration of gravity* at the equator? (Think *centripetal acceleration.*)

c.) The moon has a mass $m_m = 7.35\text{x}10^{22}\ kg$ and an orbital distance from the earth's center of $3.84\text{x}10^8$ meters. What must its orbital speed be?